5-4 The Triangle Midsegment Theorem

Warm Up

Lesson Presentation

Lesson Quiz

5-4 The Triangle Midsegment Theorem

Warm Up

Use the points *A*(2, 2), *B*(12, 2) and *C*(4, 8) for Exercises 1–5.

1. Find *X* and *Y*, the midpoints of $\overline{AC}$ and $\overline{CB}$. (3, 5), (8, 5)

2. Find *XY*. 5

3. Find *AB*. 10

4. Find the slope of $\overline{AB}$. 0

5. Find the slope of $\overline{XY}$. 0

6. What is the slope of a line parallel to $3x + 2y = 12$? $-\frac{3}{2}$

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Objective

Prove and use properties of triangle midsegments.

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Vocabulary

midsegment of a triangle

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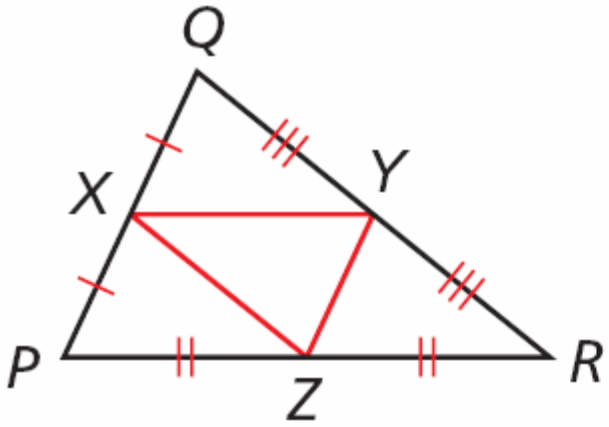
A **midsegment of a triangle** is a segment that joins the midpoints of two sides of the triangle. Every triangle has three midsegments, which form the *midsegment triangle*.



Midsegments: $\overline{XY}$, $\overline{YZ}$, $\overline{ZX}$

Midsegment triangle: $\triangle XYZ$

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Example 1: Examining Midsegments in the Coordinate Plane

The vertices of ΔXYZ are $X(-1, 8)$, $Y(9, 2)$, and $Z(3, -4)$. M and N are the midpoints of $\overline{XZ}$ and $\overline{YZ}$. Show that $\overline{MN} \parallel \overline{XY}$ and $MN = \frac{1}{2}XY$.

Step 1 Find the coordinates of M and N.

$$\text{mdpt. of } \overline{XZ} = \left(\frac{-1+3}{2}, \frac{8+(-4)}{2}\right) = (1, 2)$$

$$\text{mdpt. of } \overline{YZ} = \left(\frac{9+3}{2}, \frac{2+(-4)}{2}\right) = (6, -1)$$

Example 1 Continued

Step 2 Compare the slopes of $\overline{MN}$ and $\overline{XY}$.

$$\text{slope of } \overline{MN} = \frac{-1-2}{6-1} = -\frac{3}{5}$$

$$\text{slope of } \overline{XY} = \frac{2-8}{9-(-1)} = -\frac{3}{5}$$

Since the slopes are the same, $\overline{MN} \parallel \overline{XY}$.

Example 1 Continued

Step 3 Compare the heights of $\overline{MN}$ and $\overline{XY}$.

$$MN = \sqrt{(6-1)^2 + (-1-2)^2} = \sqrt{34}$$

$$XY = \sqrt{[9-(-1)]^2 + (2-8)^2} = 2\sqrt{34}$$

Since $\sqrt{34} = \frac{1}{2}\left(2\sqrt{34}\right)$, $MN = \frac{1}{2}XY$.

Check It Out! Example 1

The vertices of ΔRST are $R(-7, 0)$, $S(-3, 6)$, and $T(9, 2)$. M is the midpoint of $\overline{RT}$, and N is the midpoint of $\overline{ST}$. Show that $\overline{MN} \parallel \overline{RS}$ and $MN = \frac{1}{2}RS$.

Step 1 Find the coordinates of M and N.

$$\text{mdpt. of } \overline{RT} = \left(\frac{-7+9}{2}, \frac{0+2}{2}\right) = (1, 1)$$

$$\text{mdpt. of } \overline{ST} = \left(\frac{-3+9}{2}, \frac{6+2}{2}\right) = (3, 4)$$

Check It Out! Example 1 Continued

Step 2 Compare the slopes of $\overline{MN}$ and $\overline{RS}$.

$$\text{slope of } \overline{MN} = \frac{3}{2}$$

$$\text{slope of } \overline{RS} = \frac{3}{2}$$

Since the slopes are equal $\overline{MN} \parallel \overline{RS}$.

Check It Out! Example 1 Continued

Step 3 Compare the heights of $\overline{MN}$ and $\overline{RS}$.

$$MN = \sqrt{(4-1)^2 + (3-1)^2} = \sqrt{13}$$

$$RS = \sqrt{(6-0)^2 + [-3-(-7)]^2} = \sqrt{52} = 2\sqrt{13}$$

The length of $\overline{MN}$ is half the length of $\overline{RS}$.

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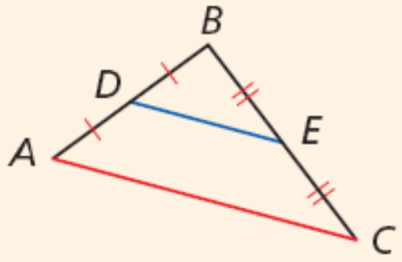
The relationship shown in Example 1 is true for the three midsegments of every triangle.

Theorem 5-4-1 **Triangle Midsegment Theorem**

A midsegment of a triangle is parallel to a side of the triangle, and its length is half the length of that side.

$\overline{DE} \parallel \overline{AC}$, $DE = \frac{1}{2}AC$



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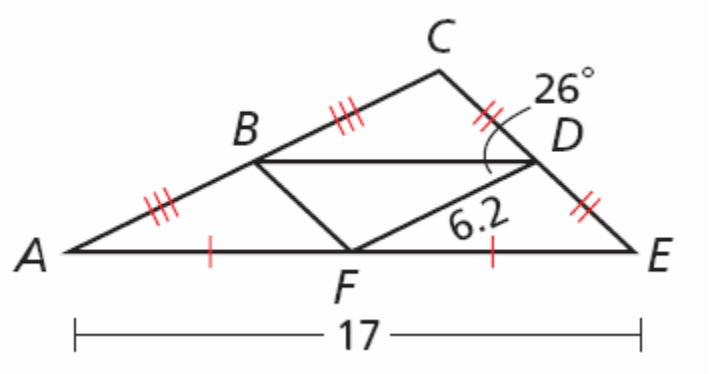
Example 2A: Using the Triangle Midsegment Theorem

Find each measure.

BD



$BD = \frac{1}{2}AE$ *Δ Midsegment Thm.*

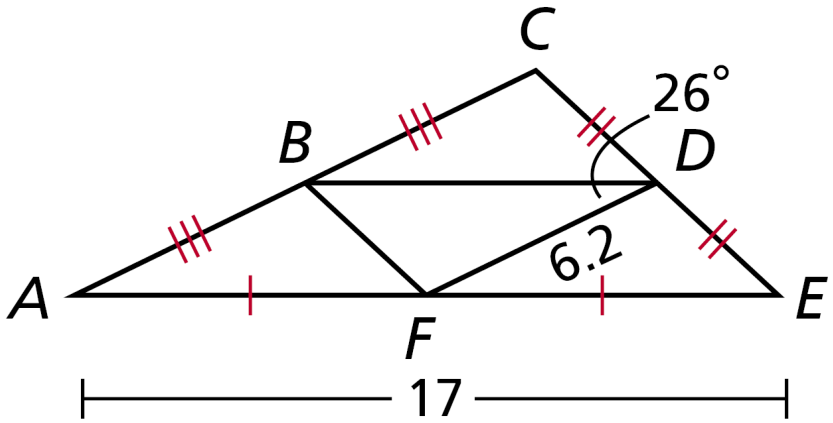
$BD = \frac{1}{2}(17)$ *Substitute 17 for AE.*

$BD = 8.5$ *Simplify.*

Example 2B: Using the Triangle Midsegment Theorem

Find each measure.

m∠*CBD*



$\overline{DF} \parallel \overline{CA}$ *Δ Midsegment Thm.*

$m\angle CBD = m\angle BDF$ *Alt. Int. ∠s Thm.*

$m\angle CBD = 26°$ *Substitute 26° for m ∠BDF.*

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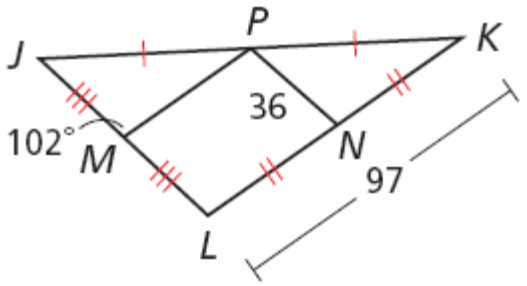
Check It Out! Example 2a

Find each measure.

JL



$PN = \frac{1}{2}JL$ *Δ Midsegment Thm.*

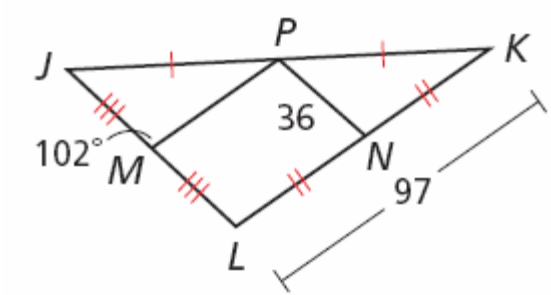
$2(36) = JL$ *Substitute 36 for PN and multiply both sides by 2.*

$72 = JL$ *Simplify.*

Check It Out! Example 2b

Find each measure.

PM



$PM = \frac{1}{2}LK$ *Δ Midsegment Thm.*

$PM = \frac{1}{2}(97)$ *Substitute 97 for LK.*

$PM = 48.5$ *Simplify.*

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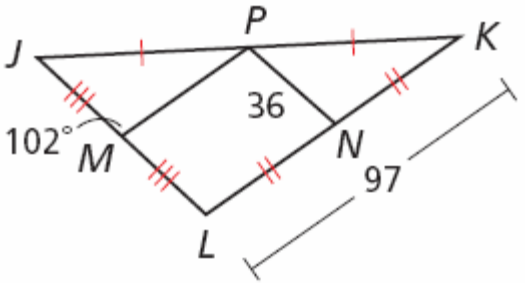
Check It Out! Example 2c

Find each measure.

m∠*MLK*



$\overline{MP} \parallel \overline{LK}$ *Δ Midsegment Thm.*

$m\angle MLK = m\angle JMP$ *Similar triangles*

$m\angle MLK = 102°$ *Substitute.*

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Example 3: Indirect Measurement Application

In an A-frame support, the distance PQ is 46 inches. What is the length of the support $\overline{ST}$ if S and T are at the midpoints of the sides?

$ST = \frac{1}{2}PQ$ *Δ Midsegment Thm.*

$ST = \frac{1}{2}(46)$ *Substitute 46 for PQ.*

$ST = 23$ *Simplify.*

The length of the support *ST* is 23 inches.

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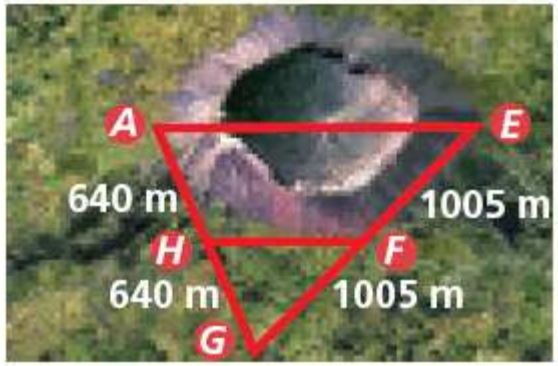
Check It Out! Example 3

What if…? Suppose Anna's result in Example 3 (p. 323) is correct. To check it, she measures a second triangle. How many meters will she measure between *H* and *F*?



$HF = \frac{1}{2}AE$ *Δ Midsegment Thm.*

$HF = \frac{1}{2}(1550)$ *Substitute 1550 for AE.*

$HF = 775$ m *Simplify.*

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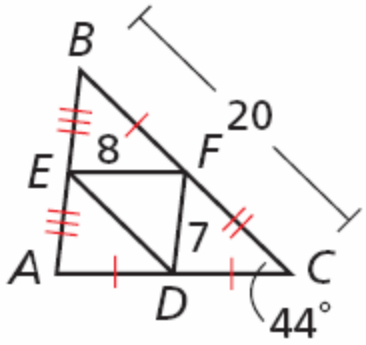
Lesson Quiz: Part I

Use the diagram for Items 1–3. Find each measure.



1. *ED* 10

2. *AB* 14

3. m∠*BFE* 44°

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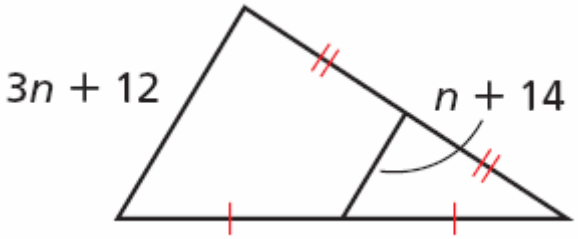
Lesson Quiz: Part II

4. Find the value of *n.*



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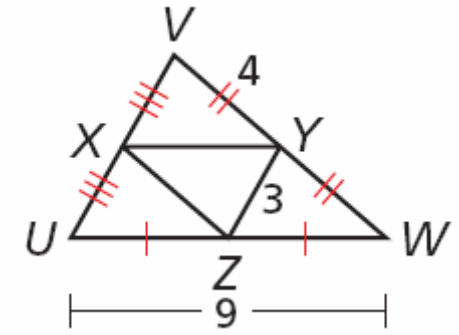
5. Δ*XYZ* is the midsegment triangle of Δ*WUV.* What is the perimeter of Δ*XYZ?*



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