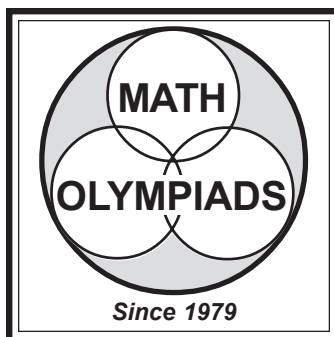


OLYMPIAD PROBLEMS

2004-2005

DIVISION E

WITH
ANSWERS AND SOLUTIONS



Mathematical Olympiads for Elementary and Middle Schools

A Nonprofit Public Foundation

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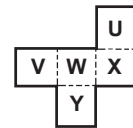
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**1A** Time: 4 minutes

What is the value of $(185 + 278 + 579) - (85 + 178 + 279)$?

1B Time: 4 minutes

A piece of paper is cut out along the solid lines and labeled as shown. It is folded along the dotted lines to make an open box. The box is placed so that the top is open. What letter is on the bottom of the box?

**1C** Time: 4 minutes

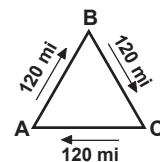
Alexis, Emma, and Li play in the school band. One plays the flute. One plays the saxophone. One plays the drums. Alexis is a 6th grader. Alexis and the saxophone player practice together after school. Emma and the flute player are 5th graders. Who plays the drums?

1D Time: 5 minutes

Avi uses 11 toothpicks to form a row of 5 attached triangles, as shown. Suppose he continues this pattern, using 89 toothpicks in all. What is the total number of triangles formed?

**1E** Time: 7 minutes

Dale travels from city A to city B to city C and back to city A. Each city is 120 miles from the other two. Her average rate from A to B is 60 mph. Her average rate from B to C is 40 mph. Her average rate from C to A is 24 mph. Overall, what is Dale's average rate for the entire trip?



**2A** Time: 4 minutes

An even number between 100 and 125 is divisible by 3 and also by 5. What is that number?

2B Time: 5 minutes

Dom has \$120 in his bank account. He deposits \$6 at the end of each week. Kim has \$200 in her account. She withdraws \$4 at the end of each week. At the end of how many weeks will they have the same amount in their accounts?

2C Time: 6 minutes

Amanda, Beth, and Sarah run three races. In each race, one of them earns 5 points, one of them earns 3 points, and one of them earns 1 point. After the three races Beth has the highest point total. What is the **smallest** total score that Beth can have?

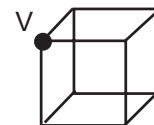
2D Time: 5 minutes

All three horizontal rows of this grid have the same sum after just two numbers are interchanged. What are the two numbers that are interchanged?

16	8	22
23	4	12
11	27	15

2E Time: 6 minutes

An ant sits at vertex V of a cube with edge of length 1 m. The ant moves along the edges of the cube and comes back to vertex V without visiting any other point twice. Find the number of meters in the length of the longest such path.



**3A** Time: 4 minutes

Points A, B, C and D are on a line. They are not necessarily in that order. Point A is between B and C. Point B is between A and D. Point D is to the left of point C. List the points in order from left to right.

3B Time: 5 minutes

Each day Jeffrey earns \$3 for doing certain chores. He can earn \$5 instead by doing additional chores as well. After ten days of doing chores, Jeffrey has earned a total of \$36. On how many of these days did Jeffrey do additional chores?

3C Time: 5 minutes

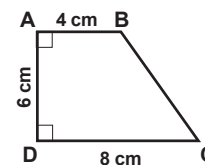
A four-digit number is written on a piece of paper. Ashley spills whiteout on it. Now the last two digits are no longer visible:

$$86??$$

The four-digit number is divisible by three, by four, and by five. Find the four-digit number.

3D Time: 6 minutes

In the diagram, $AB = 4$ cm, $AD = 6$ cm, and $CD = 8$ cm. Angles A and D are right angles. Find the area of ABCD in square centimeters.

**3E** Time: 7 minutes

Two watches are set correctly at 7:00 AM. One watch gains 3 minutes every two hours. The other watch loses 1 minute every two hours. What is the correct time the next day when the faster watch be exactly one hour ahead of the slower watch? (Indicate AM or PM.)

**4A** Time: 4 minutes

What is the smallest natural number that can be added to 259 so that the result is a multiple of 25?

4B Time: 5 minutes

A fair die is thrown. What is the probability that the top face shows a factor of 6?

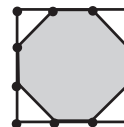
4C Time: 5 minutes

In this subtraction the boxes contain the digits 3, 4, 5, and 8 in some order and the circles contain the digits 1, 3, 7, and 9 in some order. What four-digit number is represented by the boxes?

$$\begin{array}{r} \square\square\square\square \\ - \circ\square\square\square \\ \hline 3699 \end{array}$$

4D Time: 6 minutes

Each side of the 9 cm by 9 cm square shown is divided into three equal parts. Find the area of the shaded region.

**4E** Time: 7 minutes

Colored beads are placed in the following order: 1 red, 1 green; then 2 red, 2 green; then 3 red, 3 green; and so on. In all, how many of the first 100 beads are red?

**5A** Time: 5 minutes

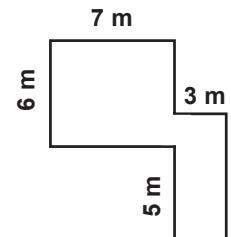
What number multiplied by itself is equal to the product of 6 and 150?

5B Time: 5 minutes

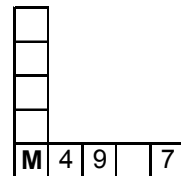
Jessie has \$5.10 worth of stamps. She has equal numbers of 50-cent, 20-cent, 10-cent, and 5-cent stamps. She has no other stamps. How many 50-cent stamps does she have?

5C Time: 6 minutes

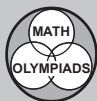
What is the perimeter in meters of the figure shown? All angles are right angles.

**5D** Time: 7 minutes

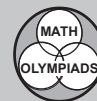
Each of the numbers 1 through 9 are placed one per square in the figure. The total of the 5 numbers in the horizontal row is the same as the total of the 5 numbers in the vertical column. Find all the different values that **M** can be.

**5E** Time: 5 minutes

A bag contains some marbles, all of the same size. Eight of them are black. The rest are red. The probability of drawing a red marble from the bag is $\frac{2}{3}$. Find the total number of red marbles in the bag.



ANSWERS AND SOLUTIONS



Note: Shaded box in upper right corner of solution indicates percent of all competitors with a correct answer.

OLYMPIAD 1

NOVEMBER 16, 2004

Answers: [1A] 500 [1B] W [1C] Alexis [1D] 44 [1E] 36

72% correct

1A METHOD 1: Strategy: Subtract in a useful way.

Note that each number in the first set is larger than its counterpart in the second set by an obvious amount. Thus:

$$185 - 85 = 100$$

$$278 - 178 = 100$$

$$579 - 279 = 300$$

Then $100 + 100 + 300 = 500$. The value is 500.

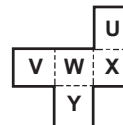
METHOD 2: Strategy: Follow the Order of Operations.

Under the order of operations we evaluate the expression in parentheses first. $(185 + 278 + 579) = 1042$. $(85 + 178 + 279) = 542$. $1042 - 542 = 500$. The value is 500.

79%

1B Strategy: Visualize the result.

When folded, V is opposite X and U is opposite Y. The remaining face, labeled W, has no face opposite it. **W is on the bottom of the box.** Cutting out a model and folding it may make the solution easier to visualize.



56%

1C Strategy: Use reasoning.

Alexis is a 6th grader. Emma and the flute player are 5th graders. Then Alexis does not play the flute, nor does Emma. Then Li plays the flute.

Alexis and the saxophone player practice together. Then Alexis does not play the saxophone. So Emma plays the saxophone and therefore **Alexis plays the drums.**

23%

1D METHOD 1: Strategy: Use reasoning.

The first three toothpicks form one triangle. After that, every two additional toothpicks form another triangle. After three toothpicks are used, 86 are left. These 43 pairs of toothpicks form 43 more triangles. Including the first triangle, **44 triangles are formed.**

METHOD 2: Strategy: Make a table of simpler cases and look for a pattern.

Number of toothpicks	3	5	7	9	11	...	89
Number of triangles	1	2	3	4	5	...	?

If we subtract 1 from the number of toothpicks and then divide by 2, the result is the number of triangles. $89 - 1 = 88$ and $88 \div 2 = 44$, so 89 toothpicks form 44 triangles. (Some students may obtain 44 triangles by finding other patterns in this table.)

FOLLOW-UP: (1) Suppose the toothpicks are used to form a row of 100 squares, one toothpick per side. How many toothpicks are needed? [301] (2) How many are needed to form a row of 100 pentagons in a similar manner? [401] (3) You may want to extend this pattern through polygons of 6, 8, 10 sides, and so on.

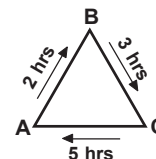
5%

1E *Strategy:* Find the time taken for each part of the trip.

The only way to find Dale's average rate for the whole trip is to divide the total distance traveled by the total time needed.

From **A** to **B** is 120 miles. Traveling at 60 mph, Dale takes $120 \div 60$ or 2 hours.

Similarly, in traveling from **B** to **C**, Dale takes $120 \div 40$ or 3 hours, and the trip from **C** back to **A** takes $120 \div 24$ or 5 hours.



The entire trip is $120 + 120 + 120$ or 360 miles. The time taken is $2 + 3 + 5$ or 10 hours. **Dale's average rate for the entire trip is $360 \div 10$ or 36 mph.**

Note: Why is the answer unequal to the average of 60, 40, and 24 mph? In general, if the times are different, then the average rate for the entire trip is different from the average of the individual rates. For example, consider an extreme case: 60 miles traveled at 2 mph followed by 60 miles traveled at 60 mph. Because the first trip alone requires 30 hours, the average rate for the entire 120 miles cannot be 31 mph.

FOLLOW-UP: The numbers of miles between cities A, B, and C are shown in the diagram. Dolly travels from A to B at 25 mph and from B to C at 10 mph. (1) How fast does she travel from C to A if her average rate for the entire trip is 16 mph? [20 mph] (2) Dolly wants her average rate for the entire trip to be 20 mph. Can she do this? Why? [No. She would have to complete the trip in a total of 6 hours, and she has already used that much time.]



OLYMPIAD 2

DECEMBER 14, 2005

Answers: [2A] 120 [2B] 8 [2C] 11 [2D] 4, 11 [2E] 8

63% correct

2A METHOD 1: *Strategy:* Find the least common multiple.

Any number divisible by 3 and 5 is divisible by their least common multiple, 15. The multiples of 15 between 100 and 125 are 105 and 120. Only 120 is even, so **the number is 120.**

METHOD 2: *Strategy:* Consider one condition at a time.

The numbers between 100 and 125 that are divisible by 5 are 105, 110, 115, and 120. Only 110 and 120 are even. Of these, only 120 is divisible by 3. The number is 120.

FOLLOW-UP: (1) Of the first 1000 natural numbers, how many are even and also divisible by 3 and 5? [33] (2) How many even natural numbers between 200 and 1000 are divisible by 3 and 5? [27]

52%

2B METHOD 1: *Strategy:* Determine the weekly decrease in the difference.

Each week Dom adds \$6 and Kim withdraws \$4, so their account totals become closer by \$10. Kim starts with 200 – 120 or \$80 more than Dom, so **at the end of $80 \div 10$ or 8 weeks they will have the same amount in their accounts.**

METHOD 2: *Strategy:* List the weekly amounts in each account.

		Amount at the end of week							
Week #	Start	1	2	3	4	5	6	7	8
Kim	200	196	192	188	184	180	176	172	168
Dom	120	126	132	138	144	150	156	162	168
Differences	80	70	60	50	40	30	20	10	0

They have the same amount at the end of 8 weeks.

METHOD 3: *Strategy:* Use algebra.

Let w = the number of weeks that pass.

Dom has $120 + 6w$ dollars and Kim has $200 - 4w$ dollars at any time.

$$120 + 6w = 200 - 4w$$

$$10w = 80$$

$$w = 8$$

At the end of 8 weeks they each have the same amount.

29%

2C *Strategy:* Find the total number of points earned.

In each race $5 + 3 + 1$ or 9 points are earned. The total number of points earned is 3×9 or 27. If all three scores are the same, each runner would have 9 points. The winning score is then greater than 9. Note also that each runner's total is the sum of three odd numbers and is therefore odd. The smallest odd number greater than 9 is 11, which is a possible winning score. For example, the scores could be 5,5,1; 3,3,3; 1,1,5. **Beth's smallest possible total score is 11.**

FOLLOW-UP: (1) 6 darts land on a dartboard. Each scores 5 or 7 or 9 points. Which score of 24, 33, 39, 46, 51, and 60 is possible? [46] (2) How many different ways can the total score of 46 be reached? [3 (777799, 577999, 559999: exchange two 7s for one 5 and one 9)]

36%

2D *Strategy:* Examine the totals of the three rows.

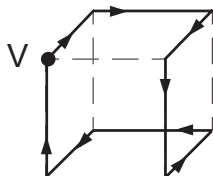
From top to bottom the totals of the three rows are 46, 39, and 53. The total of the second row is 7 less than the first row, and the total of the third row is 7 more than the first row. To make the three totals equal, increase the total of the second row by 7 and decrease the total of the third row by 7. The only two numbers in these rows that differ by 7 are 4 and 11. Once **4 and 11 are exchanged**, the total of each row becomes 46.

FOLLOW-UP: A magic square is an arrangement of numbers so that the total of each row and each column, as well as the total of the two main diagonals, is the same. Complete the following magic square inserting each odd number from 3 through 19. [From left to right the rows contain 17,3,13; 7,11,15; and 9,19,5.]

	3	
		5

50%

- 2E** *Strategy:* Trace possible paths on the cube.
Each possible path has a length of 4, 6, or 8 m. **The longest such path is 8 m.** One such path is shown below.



OLYMPIAD 3

JANUARY 11, 2005

Answers: [3A] DBAC [3B] 3 [3C] 8640 [3D] 36 [3E] 1:00 PM

74% correct

- 3A** *Strategy:* Draw a diagram.
Draw a line and place the points on it using the information given.

Statement

Diagram

1. A is between B and C.



or



2. B is between A and D.



or



3. D is to the left of C.



The points in order from left to right are **DBAC**.

66%

- 3B** **METHOD 1:** *Strategy:* Start with an extreme case.
Suppose Jeffrey did additional chores on *none* of the ten days. He would have earned \$30. This is \$6 less than he actually earned. Each day he does his additional chores he earns an additional \$2, so **he did his additional chores on 3 days.**

METHOD 2: *Strategy:* Build a table.

The first line of the table shows the number of days Jeffrey does only those certain chores and the second line shows the remainder of the ten days, in which he does additional chores. The third line shows the total amount of money he earns. Using the pattern of the third line, we can find that he did additional chores on three days.

NUMBER OF DAYS

The certain chores	0	1	2	3	...	(7)
Additional chores	10	9	8	7	...	(3)
Total \$ earned	50	48	46	44	...	36

FOLLOW-UP: Farmer Jones has chickens and goats. When he counts these animals' heads, the total is 40. When he counts these animals' feet, the total is 100. How many chickens does he have? [30]

38%

3C *Strategy:* Use the divisibility rules.

Rule

1. A number divisible by 5 ends in 5 or 0.
2. Only even numbers are divisible by 4.
3. If a number is divisible by 3, the sum of its digits is divisible by 3.
4. If a number is divisible by 4, the number formed from its last 2 digits is divisible by 4.

Possible Numbers

86_0 or 86_5
86_0
8610 or 8640 or 8670
8640

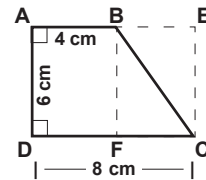
The four-digit number is 8640.

FOLLOW-UP: 8640 is also divisible by 12 and 15. Explain how you can know this without dividing. [Any number divisible by 3 and 4 is also divisible by 12. Any number divisible by 3 and 5 is also divisible by 15.]

22%

3D *Strategy:* Split the region into simpler figures.

Split the region ABCD into rectangle ABFD and triangle BFC as shown. Complete rectangle BFCE. The area of ABFD is 6×4 , or 24 sq cm. The area of triangle BFC is half the area of rectangle BFCE. Because $BF = AD$, BF is 6 cm. To find FC, observe that $DF = AB = 4$ cm, so that $FC = DC - DF$, which is $8 - 4$, or 4 cm. Since the area of BFCE = 6×4 , or 24 sq cm, the area of triangle BFC = $\frac{1}{2} \times 24$ or 12 sq cm. **The area of ABCD = $24 + 12$, or 36 sq cm.**



14%

3E *Strategy:* Determine the difference gained each hour.

Every 2 hours one watch gains 3 minutes and the other watch loses 1 minute. Then for every 2 hours the difference between their times grows by 4 minutes. For every hour the difference gained is 2 minutes. The faster watch will gain 60 minutes on the slower one in $60 \div 2$ or 30 hours. Thirty hours after 7 AM is 1:00 PM the next day. **The faster watch will be one hour ahead of the slower watch at 1:00 PM the next day.**

FOLLOW-UP: (1) Suppose the faster watch is set correctly at 7 AM and gains 4 minutes every hour. To the nearest minute, what is the correct time when the watch first shows 3:00? [2:30] (2) Suppose a watch gains 2 minutes per hour. If the watch reads correctly on Tuesday, January 13 at 7 AM, what is the next day and time in which the watch reads correctly? [Thursday, January 28 at 7 AM]

OLYMPIAD 4

FEBRUARY 8, 2005

Answers: [4A] 16 [4B] $\frac{4}{6}$ or $\frac{2}{3}$ [4C] 5438 [4D] 63 [4E] 55

62% correct

4A METHOD 1: *Strategy:* List the multiples of 25 greater than 200.

100 is a multiple of 25. Then every multiple of 100 is also a multiple of 25. Multiples of 25 greater than 200 are 225, 250, 275, 300, and so on. 275 is the smallest number that is both greater than 259 and a multiple of 25. **The smallest possible value of N is 275 – 259 or 16.**

METHOD 2: *Strategy:* Use the test for divisibility by 25.

Any natural number ending in 25, 50, 75, or 00 is a multiple of 25. The smallest such number greater than 259 is 275, which is 16 more than 259. The smallest number that can be added to 259 is 16.

Follow-Up: Suppose $259 - N$ is a multiple of 40. What is the smallest possible value of N ? [19]

38%

4B *Strategy:* Count the favorable outcomes.

The factors of 6 are 1, 2, 3, and 6. These are the four favorable outcomes. The top face could show 1, 2, 3, 4, 5, or 6, a total of six possible outcomes. All outcomes are equally likely, so the probability of a favorable outcome is the ratio of the number of favorable outcomes to the number of possible outcomes. **The probability that the top face shows a factor of 6 is $\frac{4}{6}$, or $\frac{2}{3}$. Other acceptable answers are: 4 out of 6; 2 out of 3; or $66\frac{2}{3}\%$.**

Follow-Up: A fair die is thrown. What is the probability that: (1) the top face shows a prime number? [$\frac{1}{2}$] (2) the sum of the five visible faces (top and four sides) is a prime number? [$\frac{1}{3}$]

47%

4C METHOD 1: *Strategy:* Reason from right to left.

1. From the given choices, only **D = 8** and **H = 9** can produce 9. Regrouping occurs.

$$\begin{array}{r} A\ B\ C\ \mathbf{8} \\ - E\ F\ G\ \mathbf{9} \\ \hline 3\ 6\ 9\ 9 \end{array}$$

2. Because of regrouping, **C** and **G** must be the same digit. From the choices, **C = G = 3**. Regrouping occurs.

$$\begin{array}{r} A\ B\ \mathbf{3}\ \mathbf{8} \\ - E\ F\ \mathbf{3}\ \mathbf{9} \\ \hline 3\ 6\ 9\ 9 \end{array}$$

3. Because of regrouping, either **B** or **B + 10** is 7 more than **F**. From the remaining choices, only **B = 4** and **F = 7** can produce 6. Regrouping occurs.

$$\begin{array}{r} A\ \mathbf{4}\ \mathbf{3}\ \mathbf{8} \\ - E\ \mathbf{7}\ \mathbf{3}\ \mathbf{9} \\ \hline 3\ 6\ 9\ 9 \end{array}$$

4. By elimination, **A = 5** and **E = 1**. Thus **ABCD = 5438** and **EFGH = 1739**. **The boxes represent 5438.**

$$\begin{array}{r} \mathbf{5}\ \mathbf{4}\ \mathbf{3}\ \mathbf{8} \\ - \mathbf{1}\ \mathbf{7}\ \mathbf{3}\ \mathbf{9} \\ \hline 3\ 6\ 9\ 9 \end{array}$$

METHOD 2: *Strategy:* Change subtraction to addition.

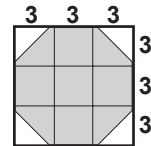
It is clearer and easier to follow the reasoning if we do the problem as an addition rather than as a subtraction. However, the reasoning is very similar in both cases.

$$\begin{array}{r} E\ F\ G\ H \\ + 3\ 6\ 9\ 9 \\ \hline A\ B\ C\ D \end{array}$$

17%

4D *Strategy:* Split the region into simpler figures.

Split the region into nine congruent squares, as shown. Each square has an area of 3×3 or 9 cm^2 .



METHOD 1: *Strategy:* Find the area that is not shaded.

The area of the large square is 9×9 or 81 cm^2 . Each of the four unshaded triangles is half of one of the small squares. The four triangles together have an area equal to two of the small squares. The unshaded area is then 2×9 or 18 sq cm , and **the area of the shaded region is $81 - 18$ or 63 sq cm .**

METHOD 2: *Strategy:* Add the areas of the smaller shaded regions.

The four shaded triangles are equal in area to two of the small squares. Combined with the five small shaded squares, the shaded region is equal in area to a total of seven small squares. The total area shaded is then 7×9 , or 63 sq cm .

28%

4E *Strategy:* Count in an organized way.

Listing of beads	Total number of red beads	Total number of all beads
RG	1	2
RG RRGG	$1+2 = 3$	6
RG RRGG RRRGGG	$1+2+3 = 6$	12
⋮	⋮	⋮
RG ... RRRRRRRRRRGGGGGGGG	$1+2+3+\dots+9 = 45$	90

The first nine groups contain 45 beads of each color. The tenth group starts with 10 red beads, and that brings the total number of beads to 100. **Of the first 100 beads, 55 are red.**

FOLLOW-UP: How many green beads are there in the first 200? [95]

OLYMPIAD 5

MARCH 8, 2005

Answers: [5A] 30 [5B] 6 [5C] 42 [5D] 1, 3 [5E] 16

57% correct

5A **METHOD 1:** *Strategy:* Find the prime factors.

$6 = 2 \times 3$ and $150 = 5 \times 30$ which can be written as $5 \times 5 \times 2 \times 3$.

Then $6 \times 150 = (2 \times 3) \times (5 \times 5 \times 2 \times 3)$. Observe that each prime factor occurs twice. Rearrange the factors: $(2 \times 3 \times 5) \times (2 \times 3 \times 5)$, which becomes 30×30 . **30 multiplied by itself is equal to the product of 6 and 150.**

METHOD 2: *Strategy:* Do the multiplication.

$6 \times 150 = 900$. Since $9 = 3 \times 3$, $900 = 30 \times 30$. The number is 30.

FOLLOW-UP: A large cube is built using all the small cubes from 25 packages. Each package contains 40 small cubes that are 1 cm on a side. How high is the large cube? [10 cm]

71%

5B METHOD 1: *Strategy:* Find the total cost of one stamp of each kind.

Imagine one “super stamp” consisting of one of each of the stamps mentioned. This super stamp costs $50 + 20 + 10 + 5$ or $85¢$. **Jessie has** $510 \div 85$ or 6 of these super stamps and therefore **6 fifty-cent stamps**.

METHOD 2: *Strategy:* Make a table

The table shows the computation of values for 1, 2, 3, ... sets of stamps.

Kind of stamp	Number of each kind					
	1	2	3	4	5	6
50-cent	50	100	150	200	250	300
20-cent	20	40	60	80	100	120
10-cent	10	20	30	40	50	60
5-cent	5	10	15	20	25	30
Total	\$0.85	\$1.70	\$2.55	\$3.40	\$4.25	\$5.10

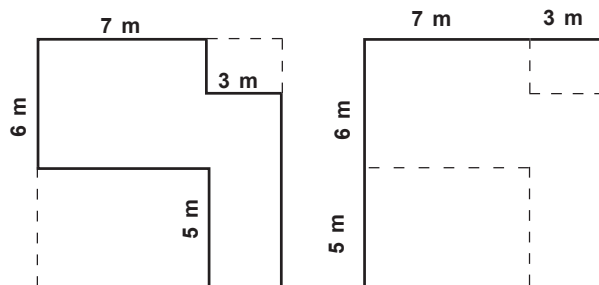
Jessie has 6 fifty-cent stamps.

FOLLOW-UP: Jessie has some $7¢$, some $13¢$, and some $37¢$ stamps. She has the same number of two kinds of these stamps and a different (larger) number of the third kind. The value of the stamps is $\$2.49$. How many of each kind does she have? [two solutions: (7 @ $7¢$, 4 @ $13¢$, and 4 @ $37¢$); also (3 @ $7¢$, 9 @ $13¢$, and 3 @ $37¢$).]

33%

5C *Strategy:* Simplify the figure.

Think of the given figure as a rectangle with two smaller rectangles cut out of it as shown on the left. Since opposite sides of a rectangle are equal in length, the given figure will have the same perimeter as the rectangle shown on the right. Since the perimeter of the rectangle is $2 \times (10 \text{ m} + 11 \text{ m})$, **the perimeter of the given figure is 42 meters.**

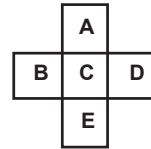


23%

5D *Strategy:* Find the possible sums of the numbers in the horizontal row.

Note first that the sum of all the numbers $1 + 2 + \dots + 9$ is 45. If the sum of the *five* numbers in the row equals the sum of the *five* numbers in the column, then the sum of the *four* numbers in each, excluding **M**, must also be equal. The sum of the four numbers in the row must be less than half of 45, or 22.5. $4 + 9 + 7 = 20$, so the sum of the four numbers in the row must be 21 or 22. This is also the sum of the four numbers in the column. All eight squares, except **M**, have a sum of either 2×21 or 2×22 . Then **M** is either $45 - 42$ or $45 - 44$. **M can be 1 or 3, only.**

FOLLOW-UP: Place 2, 3, 4, 5, and 7 in the five boxes so that the sum of B, C, and D equals the sum of A, C, and E. (1) Which values are possible for C? [3,7 only] (2) How many different ways in all can the five numbers be placed in these boxes? [16]



63%

5E METHOD 1: *Strategy:* Find the total number of marbles in the bag.

The probability of drawing a red marble is $\frac{2}{3}$, so the probability of drawing a black marble is $1 - \frac{2}{3}$, or $\frac{1}{3}$. Then 8 is $\frac{1}{3}$ of the total number of marbles. There are 3×8 or 24 marbles. Since 8 of them are black, **there are 24 – 8 or 16 red marbles in the bag.**

METHOD 2: *Strategy:* Find the ratio of the numbers of black and red marbles.

As above, the probability of drawing a black marble is $\frac{1}{3}$. The probability of drawing a red marble is twice as large, so there are twice as many red marbles as black ones. There are 2×8 or 16 red marbles in the bag.

FOLLOW-UP: Suppose the bag contains red, black, and white marbles and no other color. The probability of drawing a red marble is $\frac{1}{5}$. The probability of drawing a black marble is $\frac{1}{10}$. The bag contains 14 white marbles. How many red marbles are in the bag? [4]