Electrostatics

How is charge similar/different from mass?

“What is Charge?”

- Charge is a fundamental property of certain particles (similar to mass)
- While there is only one “type” of mass, there are two “types” of charge (positive & negative)
  - Discovered and coined by Benjamin Franklin
- SI Unit for charge is the coulomb (C)
  - Named after Charles Augustin de Coulomb
  - The variable for charge is q

"What is Charge?"

Particle	Mass	Charge
Electron	9.11×10^{-31} kg	-1.60×10^{-19} C
Proton	1.673×10^{-27} kg	$+1.60 \times 10^{-19}$ C
Neutron	1.675×10^{-27} kg	0

- Fundamental (smallest possible) charge: $e = 1.60 \times 10^{-19}$ C
- When we say an object is charged, we mean it has a "net charge" due to different numbers of protons & electrons
  - More electrons than protons: object has a <u>negative charge</u>
  - Fewer electrons than protons: object has a <u>positive charge</u>

- A chlorine ion consists of 17 protons, 18 neutrons, and 18 electrons. Determine the mass (in kg) and charge (in C) of the atom.

$$m = 5.86 \times 10^{-26} \text{ kg}$$

$$q = -1.60 \times 10^{-19} \text{ C}$$

- A sodium ion consists of 11 protons, 22 neutrons, and 10 electrons. Determine the mass (in kg) and charge (in C) of the atom.

$$m = 5.53 \times 10^{-26} \text{ kg}$$

$$q = +1.60 \times 10^{-19} \text{ C}$$

- What is the total charge of all the protons in an iron atom?

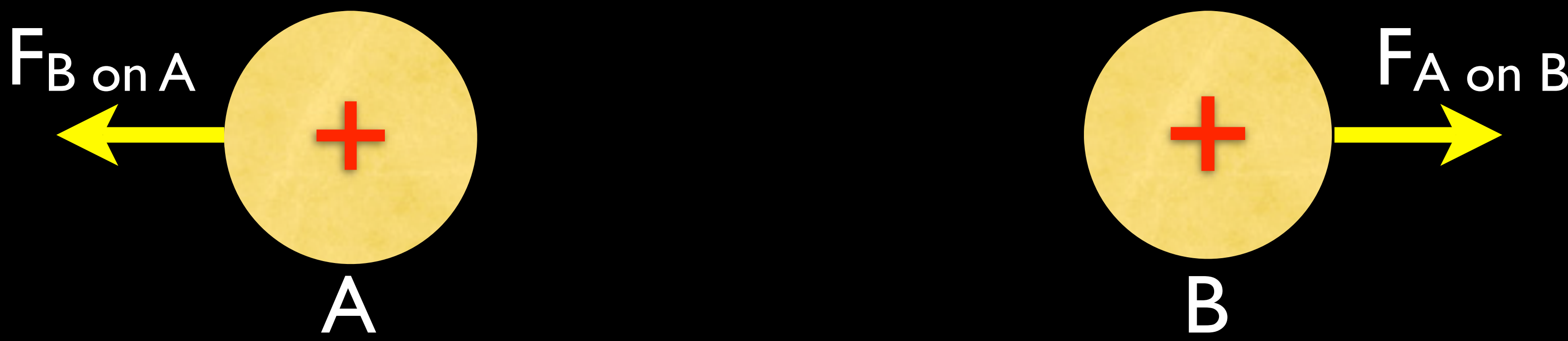
$$26 \text{ protons} \times \frac{+1.6 \times 10^{-19}\ \text{C}}{1 \text{ proton}} = +4.16 \times 10^{-18}\ \text{C}$$

- An object has a net charge of –1 coulomb. How many more electrons than protons does it have?

$$-1\ \text{C} \times \frac{1 \text{ electron}}{-1.6 \times 10^{-19}\ \text{C}} = 6.25 \times 10^{18} \text{ electrons}$$

Interacting Charged Objects
The Law of Electrostatics

- Like Charges Repel:



- Opposite Charges Attract:

Electric Force (F_E)

What determines the magnitude of this electric force?

- Recall Newton's Law of Universal Gravitation
  - F_g is directly proportional to the mass of each object and inversely proportional to the square of the distance separating them.
- F_E is directly proportional to the charge of each object and inversely proportional to the square of the distance separating them.

Coulomb's Law

$$F_E = \frac{k|q_1||q_2|}{d^2}$$

$$k = 9 \times 10^9 \,\mathrm{N}\frac{\mathrm{m}^2}{\mathrm{C}^2}$$

Tips for Using Coulomb's Law

- Be sure all distances are in meters, charges in Coulombs
  - 1 mC (millicoulomb) = 10^{-3} C
  - 1 μC (microcoulomb) = 10^{-6} C
  - 1 nC (nanocoulomb) = 10^{-9} C
- Ignore "signs" of charge (enter absolute values)
  - Use **FBFD** to determine direction of force(s)

Examples

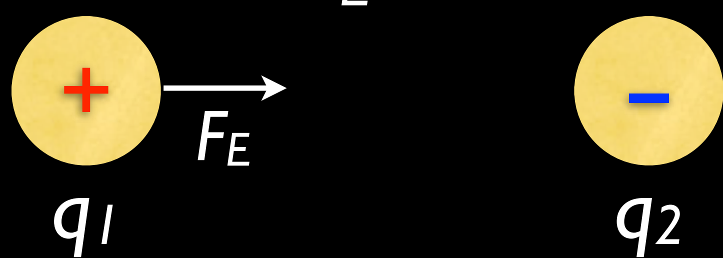
- A +3-nC point charge is placed 0.3 m to the left of a –6-nC point charge. What is the magnitude and direction of the electric force acting on the positive charge?

$|q_1| = 3 \times 10^{-9}$ C
$|q_2| = 6 \times 10^{-9}$ C
$d = 0.3$ m

$$F_E = \frac{k|q_1||q_2|}{d^2}$$

$$F_E = \frac{(9 \times 10^9)(3 \times 10^{-9})(6 \times 10^{-9})}{(0.3 \text{ m})^2}$$

$$F_E = 1.8 \times 10^{-6} \text{ N, to the right}$$



What is the magnitude and direction of the electric force acting on the negative charge?

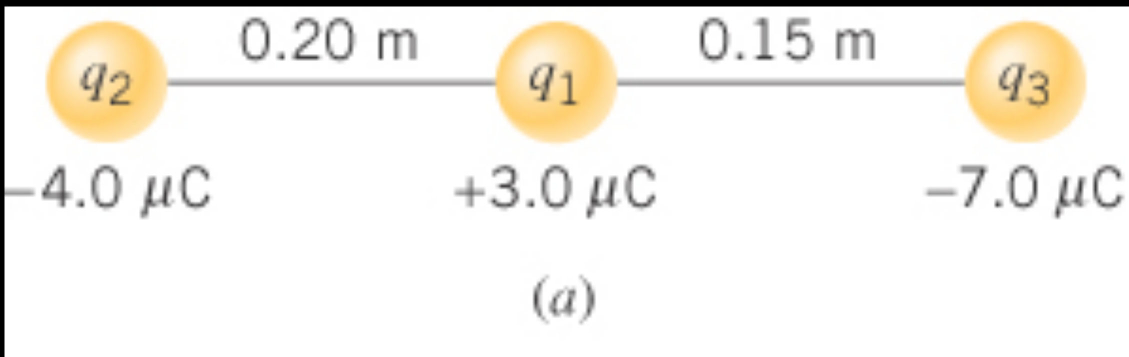
Examples

- Two charges are separated by 3.0 cm. Object A has a charge of +6.0 μC, while object B has a charge of +3.0 μC. What is the magnitude and direction ("attract" or "repel") of the electric force acting on object A?

- In the Bohr model of the hydrogen atom, the electron orbits the nuclear proton at a distance of 5.29×10^{-11} m. Determine the magnitude of the electric force the proton exerts on the electron.

Examples

- Determine the magnitude and direction of the net electrostatic force on q_1.



$F_{2\ on\ 1}$ = 2.70 N

$F_{3\ on\ 1}$ = 8.40 N

$\sum F_{on\ 1}$ = 5.70 N, Right

$\sum F_{on\ 2}$ = 0.643 N, Right

$\sum F_{on\ 3}$ = 6.34 N, Left

Transferring Charge

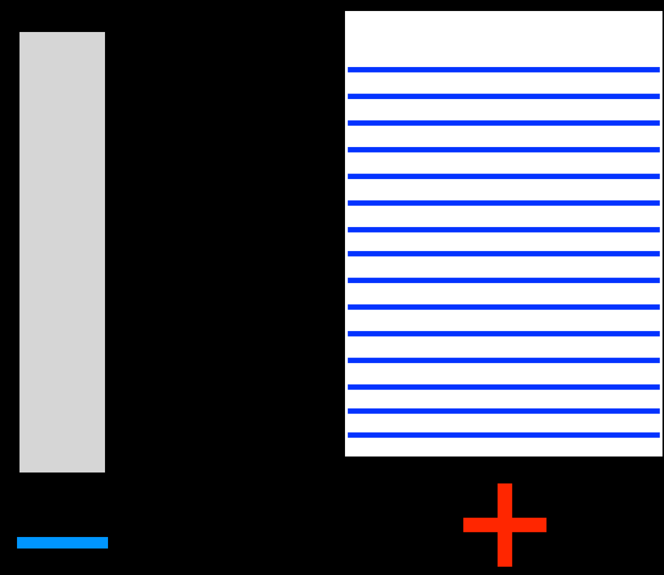
- For a neutral object to become charged it must gain or lose electrons
- Conductivity: ability of charge to move through and be transferred by a material (based on behavior of valence electrons)
  - Electrical **conductors** have a high conductivity
    - ex: metals (copper, gold, aluminum, silver, etc.)
  - Electrical **insulators** have a low conductivity
    - ex: rubber, plastics, paper, wood
  - **Semiconductors** have a variable conductivity
  - **Superconductors** have an extremely high conductivity

How Objects Become Charged

- Three methods for charging objects:
  - Friction
  - Conduction
  - Induction

Friction

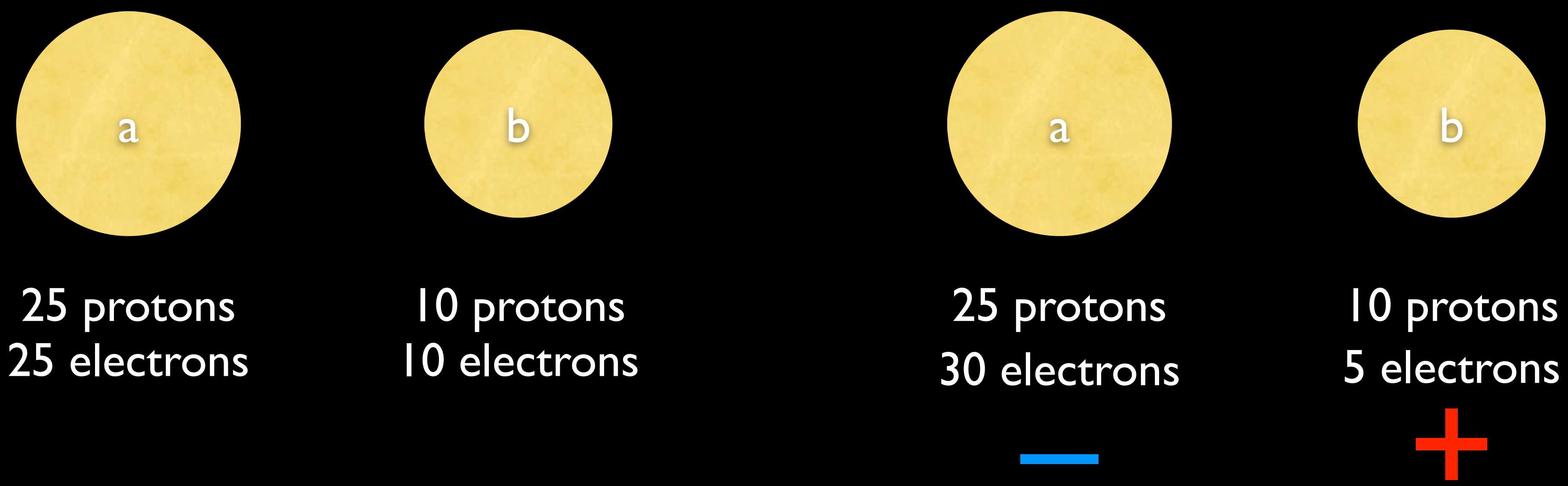
- Friction causes electrons to transfer between the two materials (even if both are insulators)
    - Lucite (clear plastic) and paper: electrons transfer to lucite



- Direction of electron transfer depends upon the relative conductivity of the two materials

Friction

- <u>Conservation of Charge</u>
  - Charge is neither created nor destroyed, just transferred from one object to another.



Conduction

- Touching a charged object to another object (neutral or charged)
  - Works best when at least one object is a good conductor
- Electrons spread out evenly over surface areas of the objects
  - More electrons end up on object with larger surface area
  - If both objects have the same surface area, the overall charge will split evenly between them

Conduction

- A metal sphere has a net charge of -4 μC. A second identical metal sphere is initially neutral. If the spheres are touched together, how much charge does each sphere end up with? How many electrons are transferred from the first sphere to the second?

Both objects will end up with a charge of -2 μC

$$-2\ \mu C \times \frac{1 \text{ electron}}{-1.6 \times 10^{-19}\ C} = 1.25 \times 10^{13} \text{ electrons}$$

Conduction

- A metal sphere has a net charge of +4 μC. A second identical metal sphere has a net charge of –6 μC. If the spheres are touched together, how much charge does each sphere end up with? How many electrons are transferred from the second sphere to the first?

The total net charge of the system is –2 μC.

Both objects will end up with a charge of <u>–1 μC</u>

The positive sphere gained -5 μC of electrons.

$$-5\ \mu C \times \frac{1\ \text{electron}}{-1.6 \times 10^{-19}\ C} = 3.125 \times 10^{13}\ \text{electrons}$$

Conduction

- Special Case: Grounding
    - Touching a charged object to an object much larger than it
    - Larger object takes away all excess electrons or adds lost electrons to neutralize object

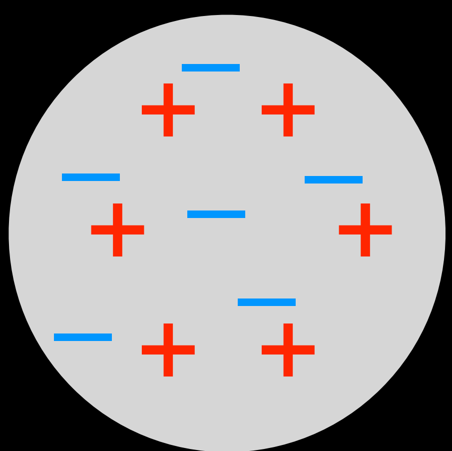
Induction

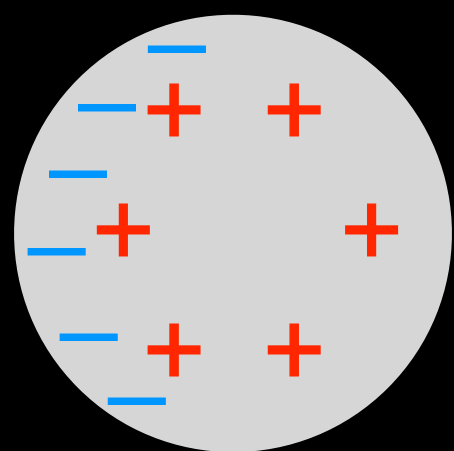
- Using a charged object to cause another object to become charged without touching them together.
  - induce: to move by persuasion or influence

- Induction is a two step process!

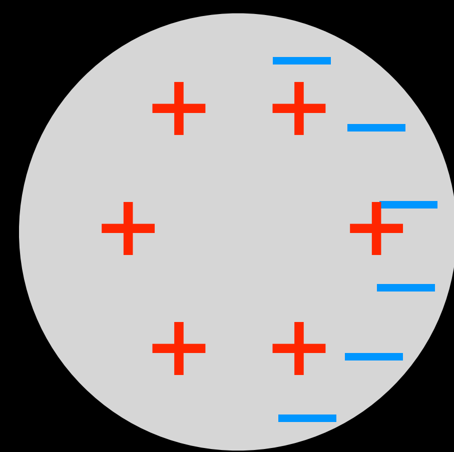
Induction

- Step One: Polarization
  - Bringing a charged object near a neutral conductor causes the electrons in that object to migrate to one side.





Positive Object to Left



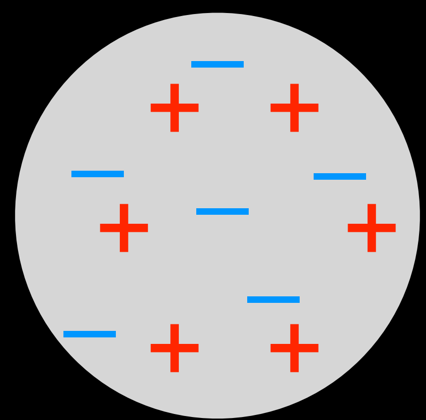
Negative Object to Left

Induction

- Step Two: Electron Transfer
  - Electrons move between polarized object and some other object through conduction

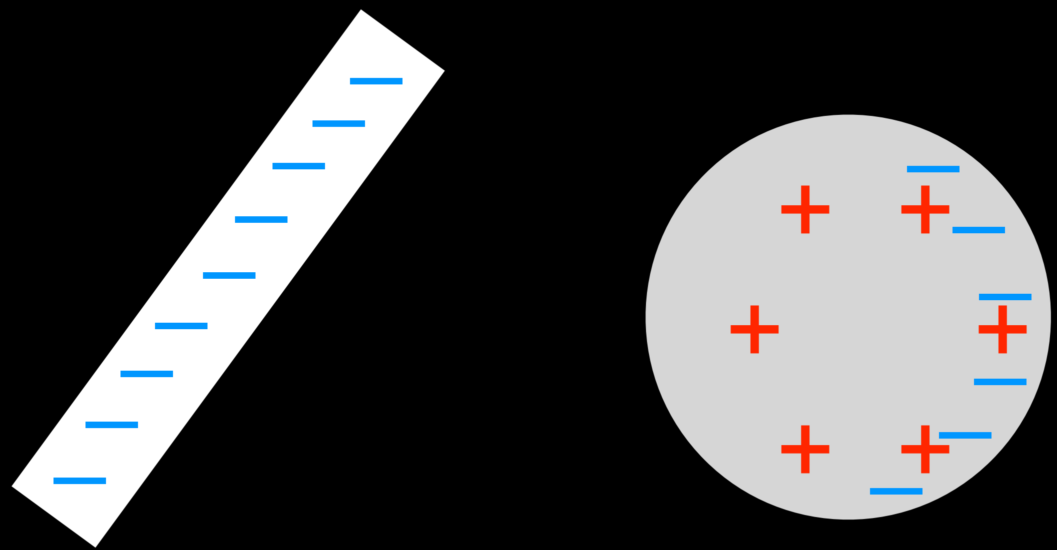
Induction

Example: Induction with Negative Object



Induction

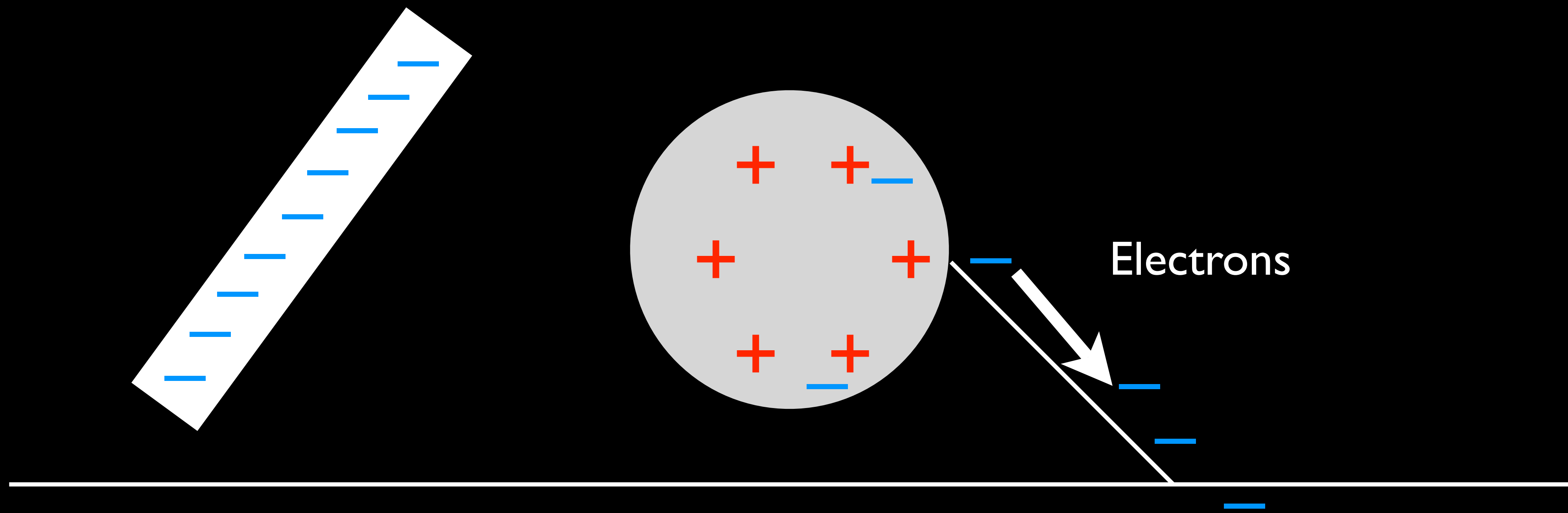
Example: Induction with Negative Object



Step 1: Polarization

Induction

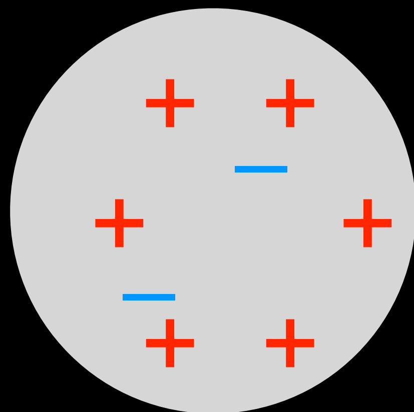
Example: Induction with Negative Object



Step 2: Electron Transfer

Induction

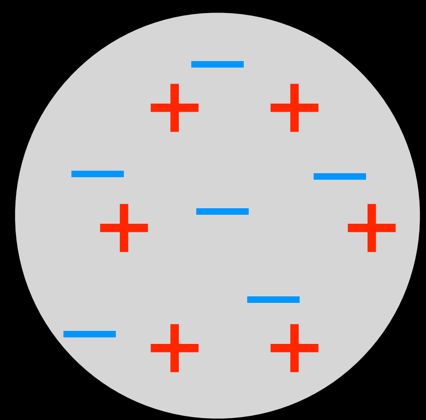
Example: Induction with Negative Object



Result: Net Positive Charge

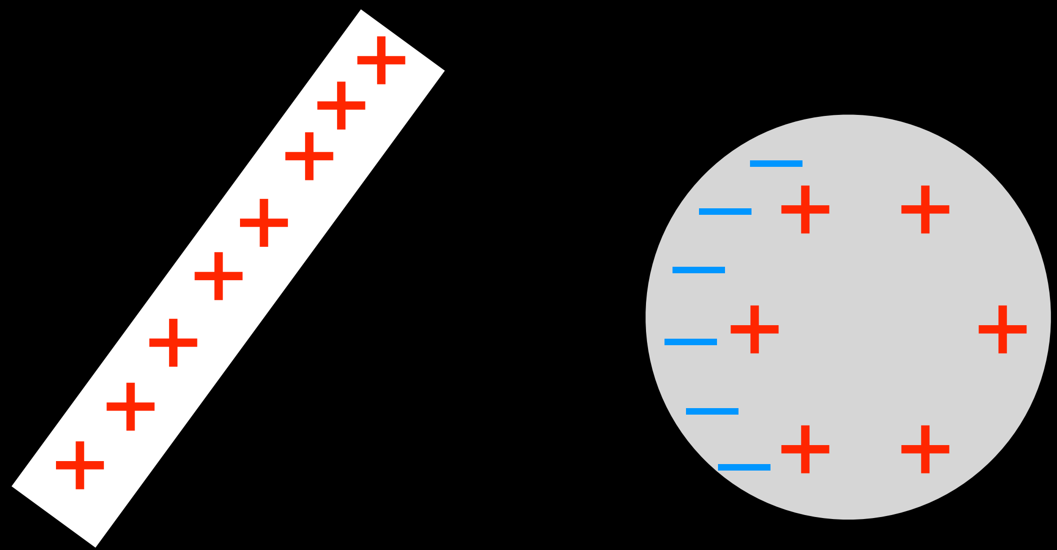
Induction

<u>Example</u>: Induction with Positive Object



Induction

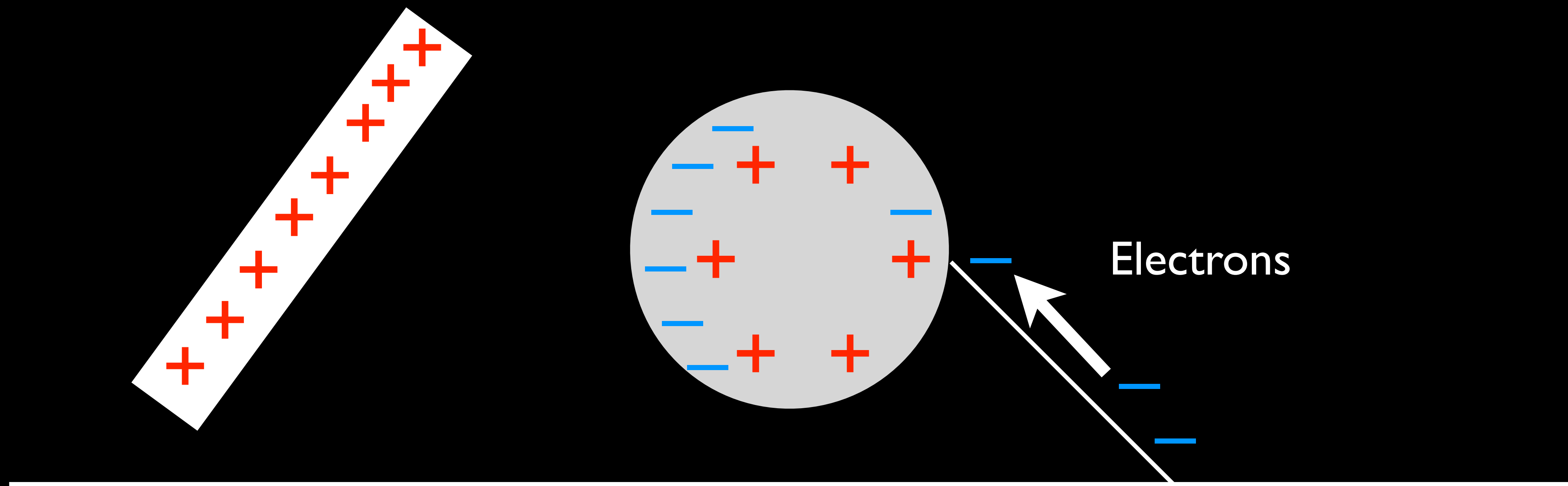
Example: Induction with Positive Object



Step 1: Polarization

Induction

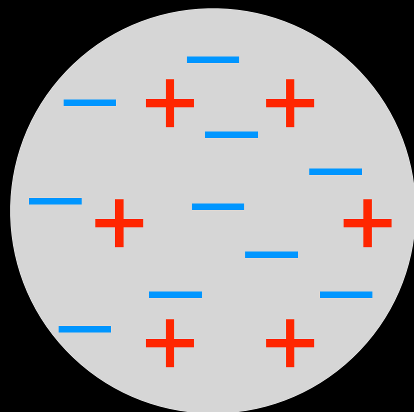
Example: Induction with Positive Object



Step 2: Electron Transfer

Induction

Example: Induction with Positive Object



Result: Negative Charge

3 Ways to Charge Objects

- Friction: objects end up with opposite charges (equal amounts)
- Conduction: objects end up with same charge
- Induction: objects end up with opposite charge

- DO NOW:
  - Calculate the force acting on Object B.

$$F_E = \frac{(9 \times 10^{9})(3 \times 10^{-6})(8 \times 10^{-6})}{(0.25)^2} = 3.46\ \text{N}$$

How does Object A exert a force at a distance?

How does Object B know Object A is there?

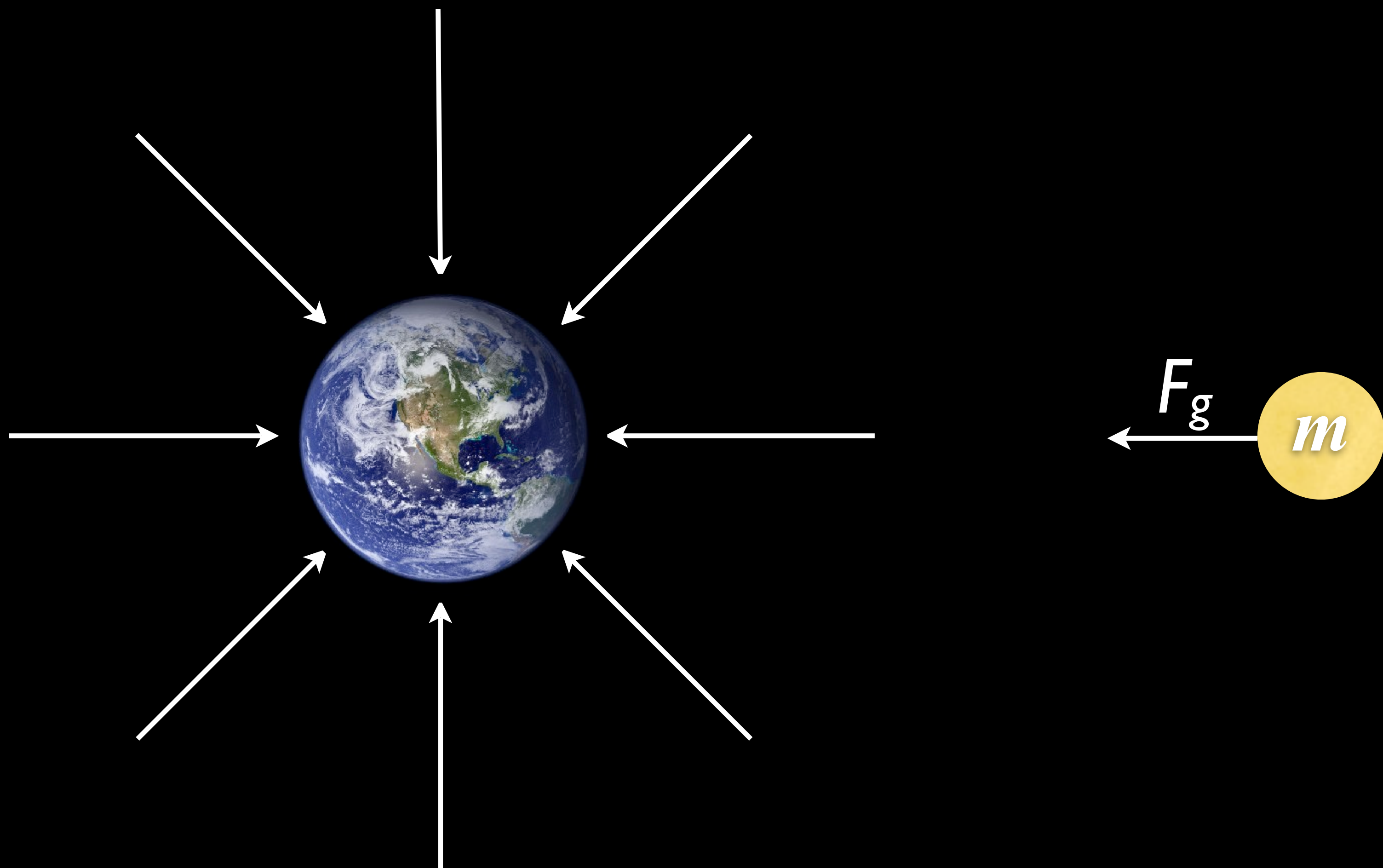
Electric Fields

- Michael Faraday (1791-1867)
  - Charged objects alter the space around them, creating an electric field (*E*).
  - When another charged object enters this electric field, it experiences a force.
- Analogous to gravity
  - Earth's mass alters the space around it, creating a gravitational field (*g*).
  - This gravitational field exerts a force on other objects with mass.

Electric Fields

Earth creates a *g* field in the space around it.

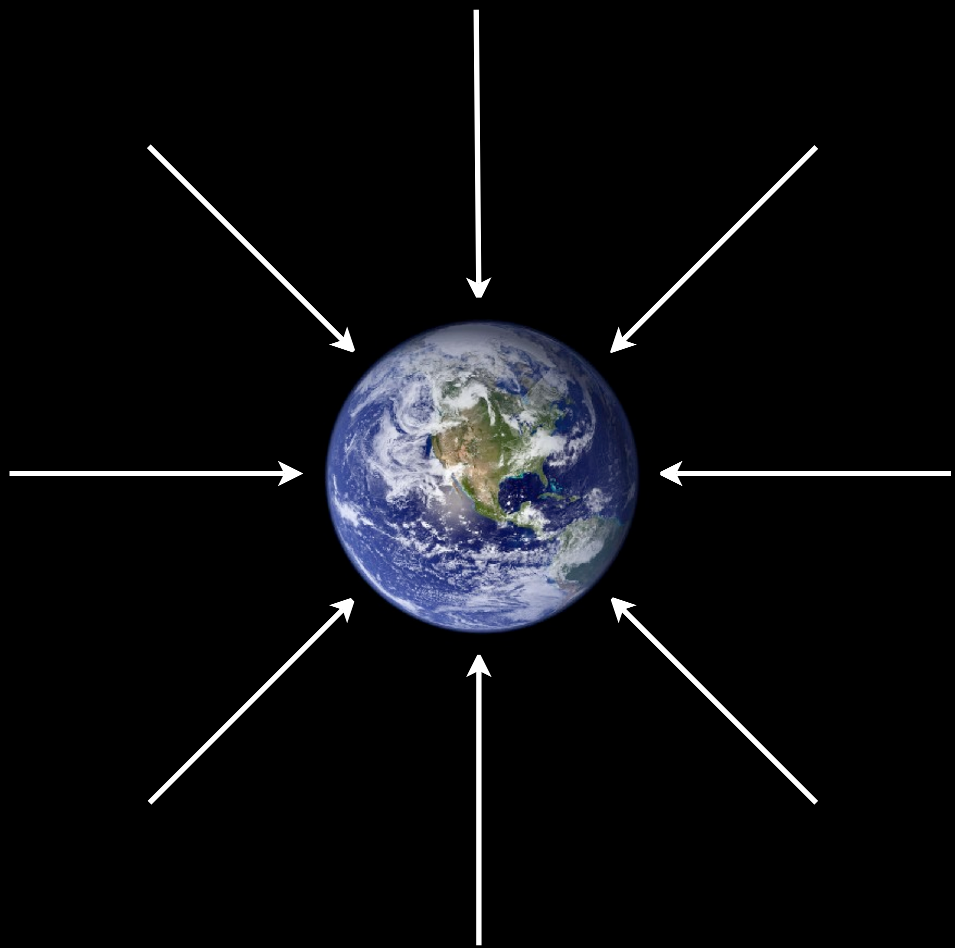
m feels a force exerted on it by the field.



Direction of *g* field vector = Direction of F_g

Electric Fields

Earth creates a g field in the space around it.



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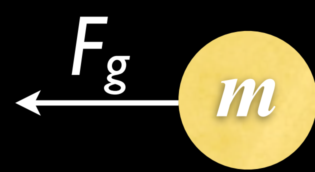


Gravitational Force exerted by Gravitational Field

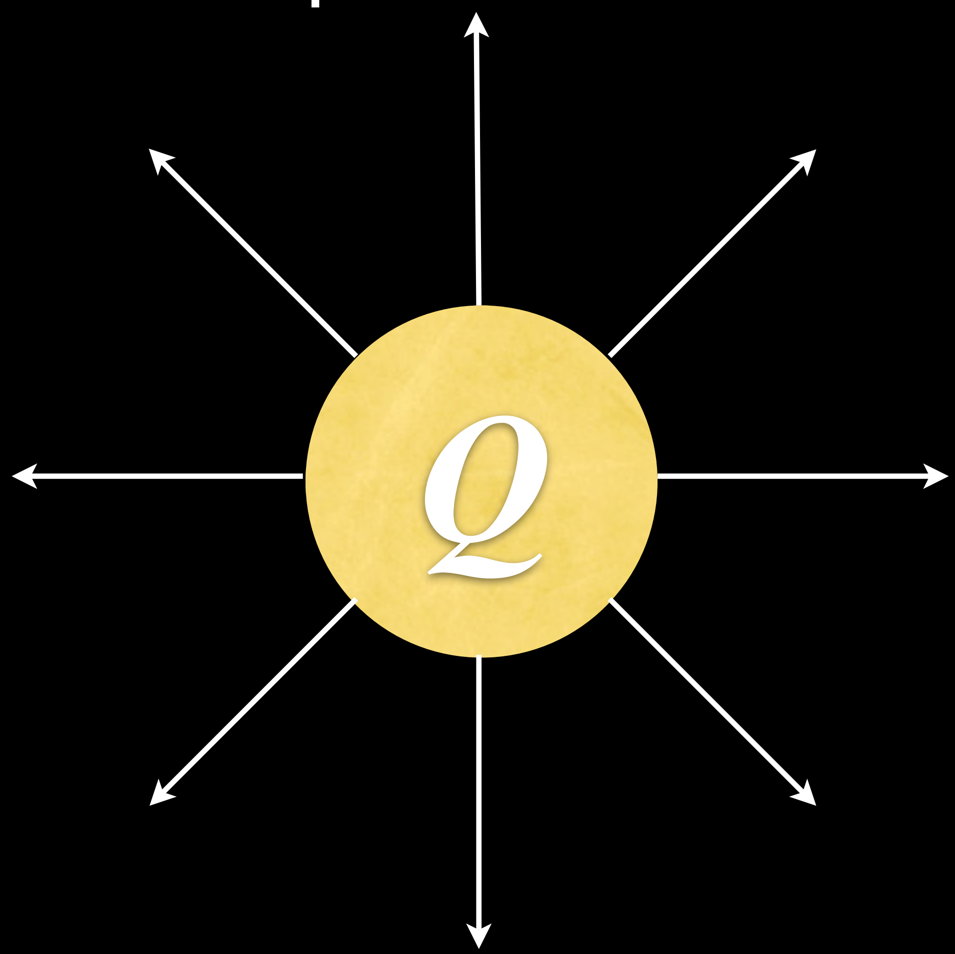
$$\vec{F}_g = m\vec{g}$$

Units for g: N/kg

Electric Fields

Q creates an *E* field in the space around it.

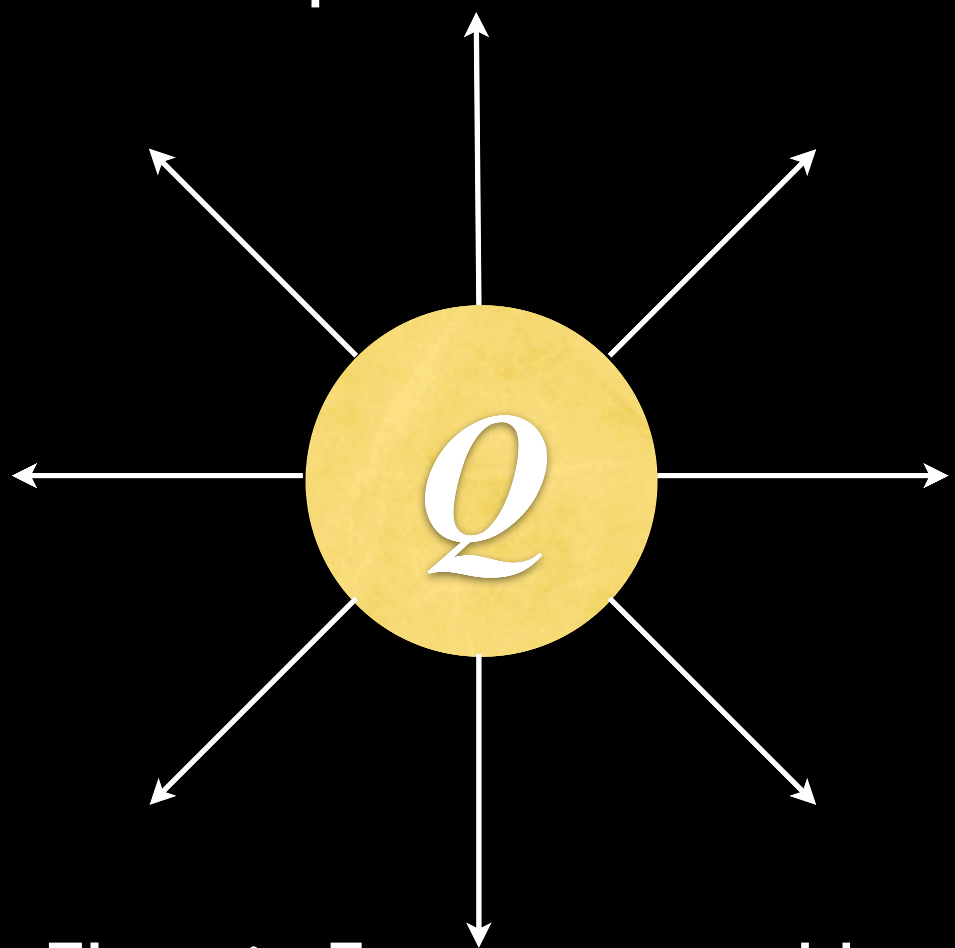
q feels a force exerted on it by the field.



Direction of *E* field vector = Direction of F_E <u>if *q* is positive</u>

Electric Fields

Q creates an *E* field in the space around it.



Electric Force exerted by Electric Field

q feels a force exerted on it by the field.

$$\vec{F}_E = q\vec{E}$$

Units for *E*: N/C

Electric Fields

- Electric Field Tips:
  - Strength of electric field may vary with location
  - Strength of electric field at a specific location is independent of *q*
  - Electric field is a vector; field vector is in the direction a positive charge would be pushed or pulled
    - A negative charge would feel a force in the exact opposite direction

Electric Fields

- Practice Problem: An object with a charge of +1.8 μC is placed in an electric field of 2.0 N/C directed to the right. Determine the magnitude and direction of the force exerted on the charge by the field.

$q = +1.8 \times 10^{-6}$ C

$E = 2.0$ N/C

$F_E = qE$

$= (1.8 \times 10^{-6}\ \text{C})(2.0\ \text{N/C})$

$= 3.6 \times 10^{-6}$ N to the right

Because the charge is positive, the force is in the same direction as the electric field.

Electric Fields

- Practice Problem: An object with a charge of –2.4 μC is placed in an electric field of 2.0 N/C directed to the right. Determine the magnitude and direction of the force exerted on the charge by the field.

$q = 2.4 \times 10^{-6}$ C

$E = 2.0$ N/C

$F_E = qE$

$= (2.4 \times 10^{-6}\text{ C})(2.0\text{ N/C})$

$= 4.8 \times 10^{-6}$ N to the left

Because the charge is negative, the force is in the direction opposite the electric field.

Comparing Mass and Charge

	Mass	Charge
Variable / SI Unit	m / kg	q / C
Fundamental Force	Gravity (F_g)	Electrostatic (F_E)
Direction of Force	Always attractive	"Likes" repel; "opposites" attract
Calculating Force	$F_g = \frac{Gm_1m_2}{d^2}$	$F_E = \frac{k\|q_1\|\|q_2\|}{d^2}$
Field	g (N/kg)	E (N/C)

Electric Fields

- Combining Definition of Electric Field and Coulomb's Law:

$$F_E = F_E$$

$$qE = \frac{k|q||Q|}{d^2}$$

Electric Field of point charge *Q*

$$E = \frac{k|Q|}{d^2}$$

Electric Fields

- Practice Problem: Find the magnitude and direction of the electric field at a distance of 10 cm from an electron.

$$E = \frac{k|Q|}{d^2} = \frac{(9 \times 10^9)(1.6 \times 10^{-19})}{(0.1)^2} = 1.44 \times 10^{-7} \text{ N/C}$$

toward the electron

Remember: E field points in direction a positive charge would be pushed or pulled.

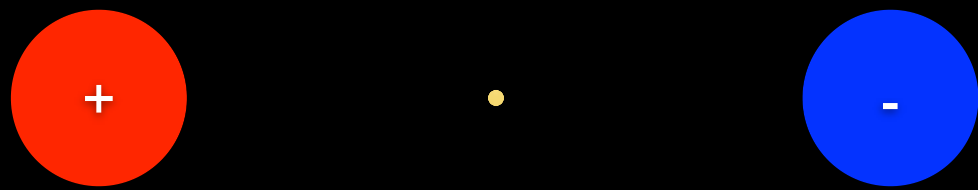
- Determine the strength of the electric field created by a +1μC point charge at a distance of (a) 1 m, (b) 2 m, and (c) 3 m.

$$E = \frac{k|Q|}{d^2} = \frac{(9 \times 10^9)(1 \times 10^{-6})}{(1)^2} = 9000 \text{ N/C}$$

$$E = \frac{k|Q|}{d^2} = \frac{(9 \times 10^9)(1 \times 10^{-6})}{(2)^2} = 2250 \text{ N/C}$$

$$E = \frac{k|Q|}{d^2} = \frac{(9 \times 10^9)(1 \times 10^{-6})}{(3)^2} = 1000 \text{ N/C}$$

- A +1μC point charge and a -1μC point charge are 0.6 m apart. Determine the magnitude and direction of the electric field at the midpoint between them.



Because electric field is a vector quantity, we can combine multiple fields using vector addition.

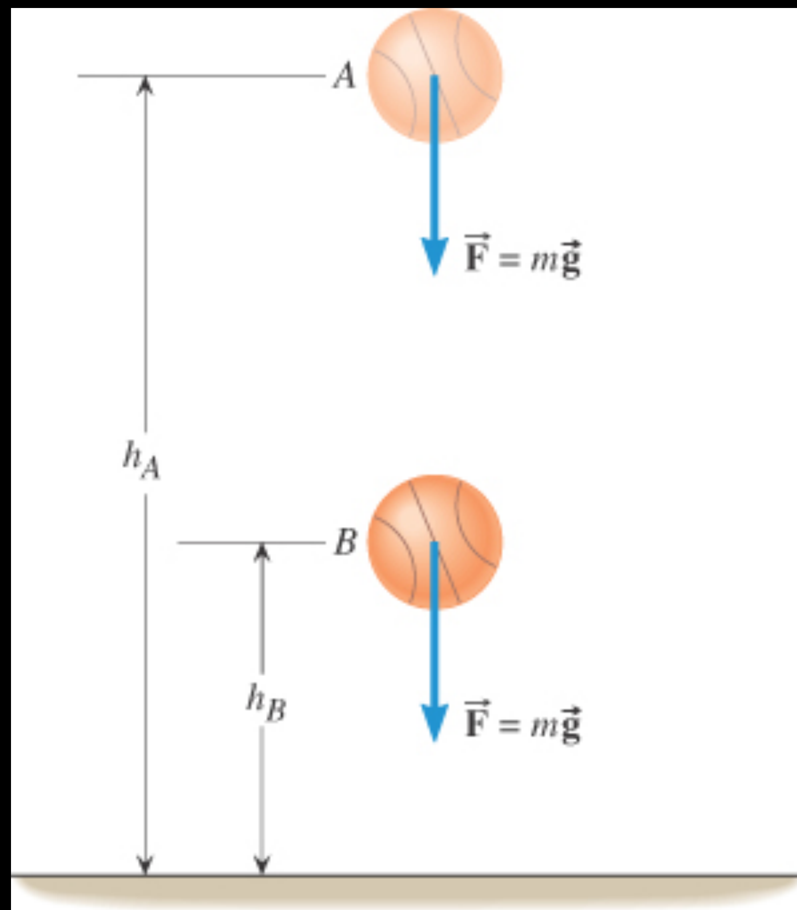
$$E_{+1\mu C} = \frac{(9 \times 10^{9})(1 \times 10^{-6})}{(0.3)^2} = 10^5 \text{ N/C} \quad \text{to the right}$$

$$E_{-1\mu C} = \frac{(9 \times 10^{9})(1 \times 10^{-6})}{(0.3)^2} = 10^5 \text{ N/C} \quad \text{to the right}$$

$$\sum E = 10^5 \text{ N/C} + 10^5 \text{ N/C} \quad = 2 \times 10^5 \text{ N/C, right}$$

Electric Potential Energy

- F_E is a conservative force (just like F_g)
  - There is a potential energy associated with F_E
- Recall Gravitational Potential Energy (*GPE*):
  - Work done by gravity ($Fd\cos\theta$) converts *GPE* into *KE* as ball moves from *A* to *B*
  - To move ball from *B* to *A* requires you to do an amount of work equal to the increase in the ball's *GPE* = $mg(h_A - h_B)$



Electric Potential Energy

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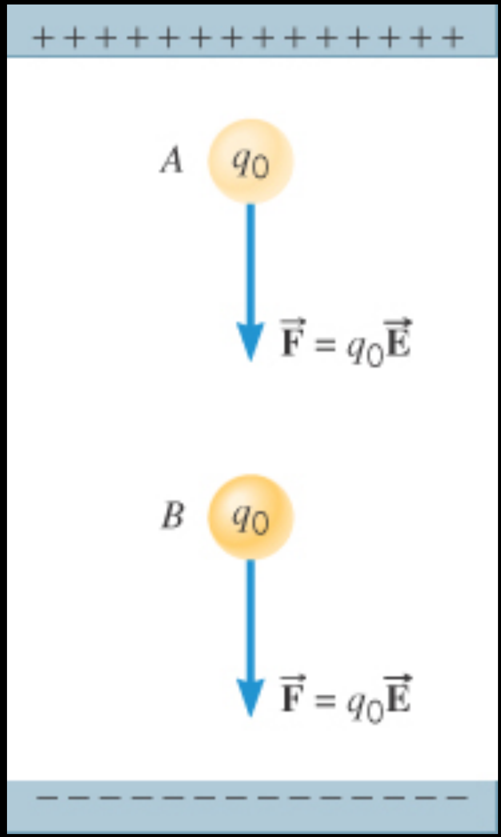


- Electric Potential Energy (*EPE*):
  - Work done by an electric field (*Fd*cosθ) converts *EPE* into *KE* as ball moves from *A* to *B*
  - To move object from *B* to *A* requires you to do an amount of work equal to the increase in its *EPE*
  - In a uniform *E* field the force is constant and:

Work done by Uniform E field $W = qEd$

Work done by Uniform E field $W = qEd$

- A charged particle is placed in an electric field of magnitude 1500 N/C. The particle moves a displacement of 1.0 cm (parallel to the electric field) due to the electric field. How much work does the field do on the particle if its charge is:

(a) 1 μC — $W = 1.5 \times 10^{-5}$ J
(b) 2 μC — $W = 3.0 \times 10^{-5}$ J
(c) 3 μC — $W = 4.5 \times 10^{-5}$ J
(d) 4 μC — $W = 6.0 \times 10^{-5}$ J

How much work does it take per coulomb?

In all cases, the field does 15 J of work per coulomb

Electric Potential Difference

- Electric potential difference between two locations is the work required to move a charge between two locations per coulomb

Definition of Electric Potential Difference

$$\Delta V = \frac{W}{q}$$

Units:

Joules per Coulomb = J/C = Volt (V)

- Example: A particle with a charge of $+8 \times 10^{-3}$ C is displaced in an electric field. The field does 1.0 J of work in moving the charge. What is the potential difference between its starting and ending locations?

$q = 8 \times 10^{-3}$ C
$W = 1.0$ J

$$\Delta V = \frac{W}{q}$$

$$= \frac{1.0\ \text{J}}{8 \times 10^{-3}\text{C}}$$

$$= 125\ \text{V}$$

- A standard AA-battery has a potential difference of 1.5 V between each end. How much energy does the battery give to each electron that it moves from the positive end to the negative end?

Remember: Work = Transfer or Conversion of Energy

$\Delta V = 1.5\,\text{V}$

$q = 1.6 \times 10^{-19}\,\text{C}$

$$\Delta V = \frac{W}{q}$$

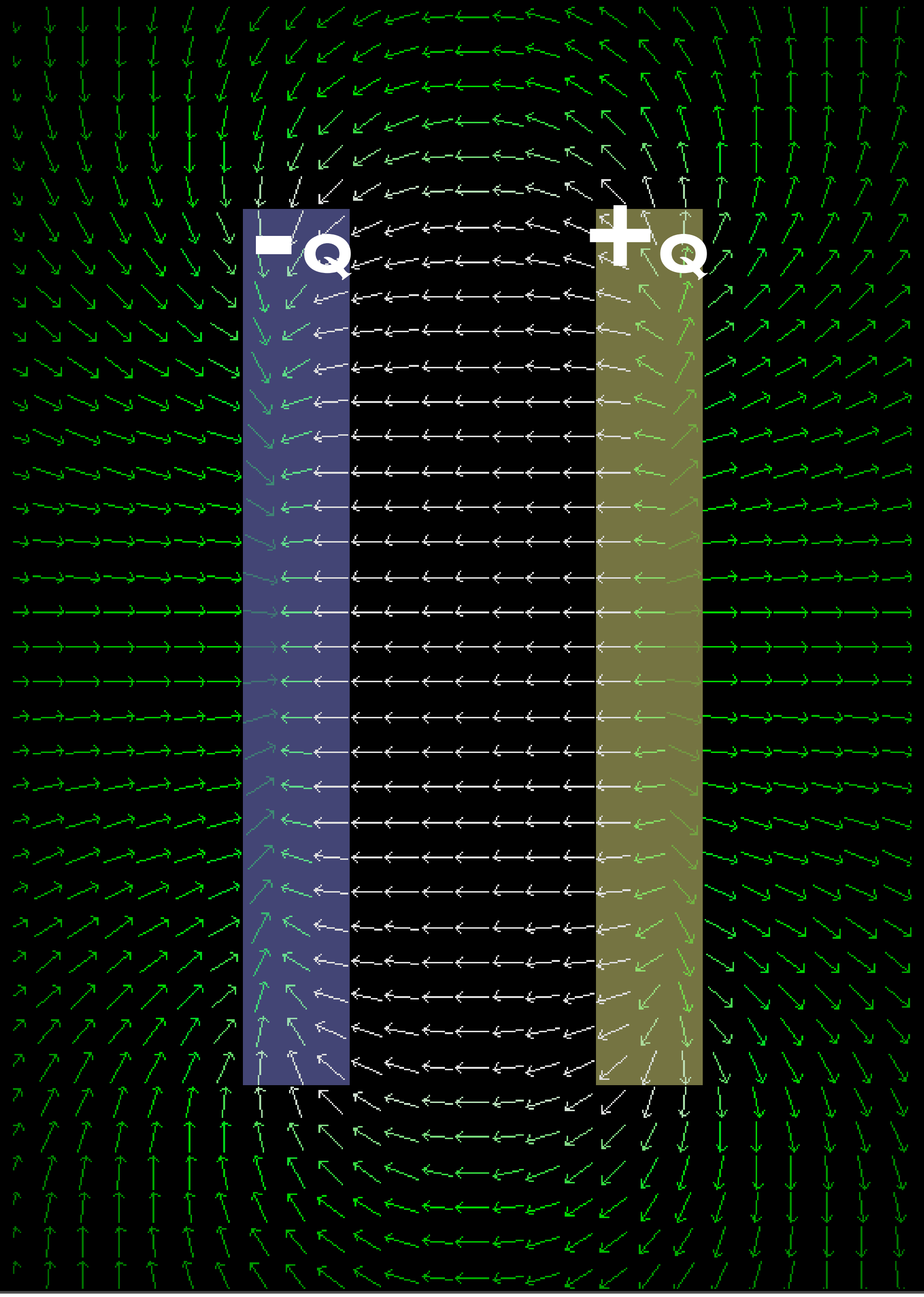
$$W = \Delta V q$$

$$= (1.5\,\text{V})(1.6 \times 10^{-19}\,\text{C})$$

$$= 2.4 \times 10^{-19}\,\text{J}$$

Parallel Plates

- The electric field (E) between two oppositely charged plates is uniform.
- A positive charge q near the + plate will experience a constant force pushing it towards the – plate.
- If the plates are separated by distance d, then: $W = qEd$



Potential Difference (Parallel Plates)

$$\Delta V = Ed$$

Parallel Plates

- Example: The electric potential difference between two oppositely charged parallel plates is 60 V. If the plates are separated by 3.0 cm, what is the magnitude of the electric field between them?

$$\Delta V = Ed$$

$$60\text{ V} = E\ (0.03\text{ m})$$

$$2000\text{ N/C} = E$$

Electric Field can also be expressed in units of volts per meter (V/m)

Parallel Plates

- Example: Determine the force felt by the charge if q = +2 μC and the potential difference between the plates is 25 V.

$$\Delta V = Ed$$

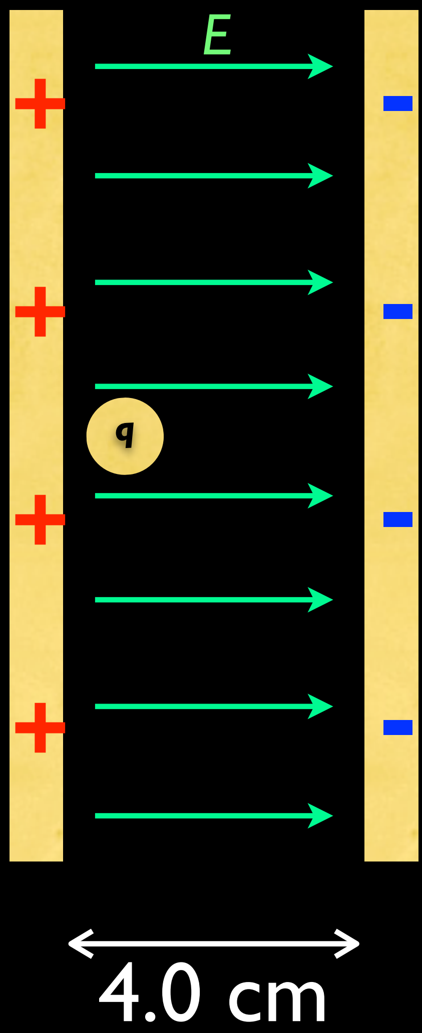
$$25\text{ V} = E\,(0.04\text{ m})$$

$$625\text{ N/C} = E$$

$$F = qE$$

$$= (2 \times 10^{-6}\text{ C})(625\text{ N/C})$$

$$= 1.25 \times 10^{-3}\text{ N}$$

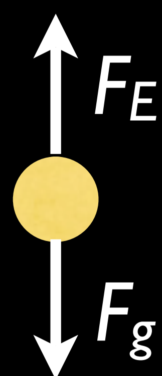


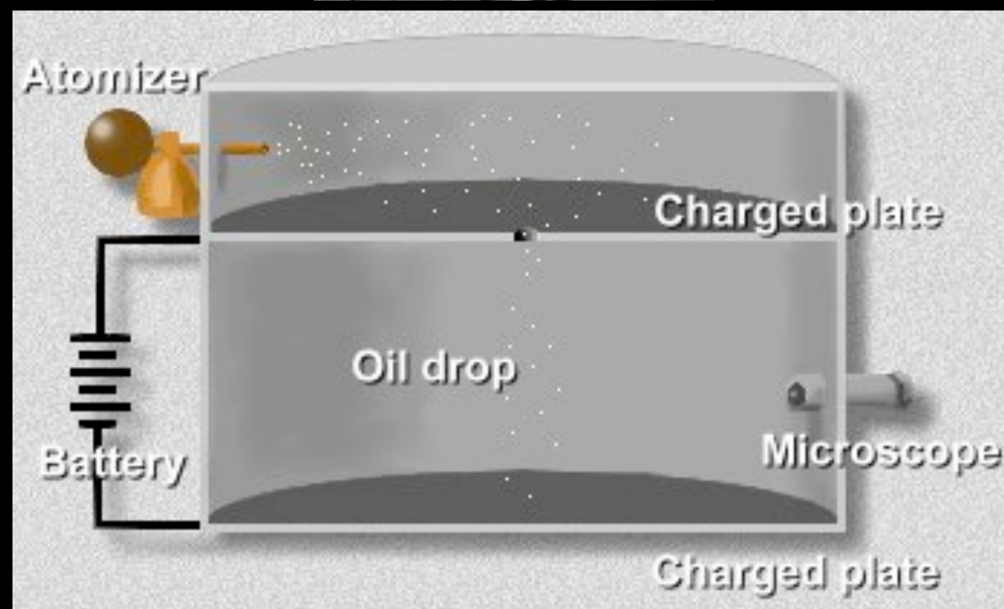
Millikan's Oil Drop Experiment

- Oil drops (charged via X-rays) drop between two metal plates.
  - Mass of oil drop determined from terminal velocity (Stokes' Law)
- Potential difference between plates adjusted until drop was in equilibrium.

$$F_g = F_E$$

$$mg = qE$$





Millikan's Oil Drop Experiment

- Example: An oil drop of mass 1.94×10^{-15} kg is held in equilibrium between two parallel plates (ΔV = 792 V). If the plates are separated by 2 cm, what is the charge on the oil drop?

$$F_g = F_E$$

$$mg = qE$$

$$mg = q(\Delta V/d)$$

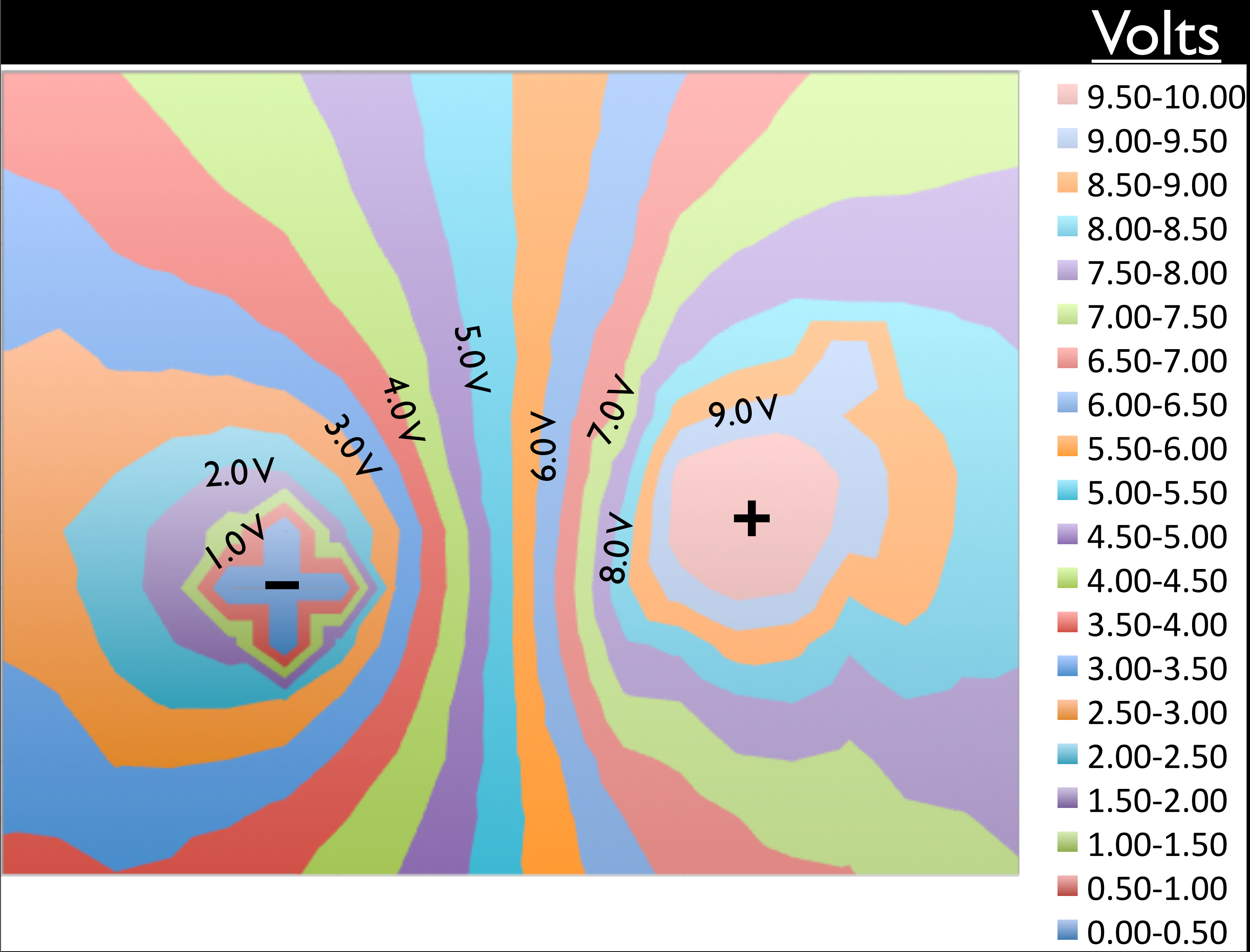
$$\frac{mgd}{\Delta V} = q$$

$$4.80 \times 10^{-19}\ \text{C} = q$$

Millikan's Oil Drop Experiment

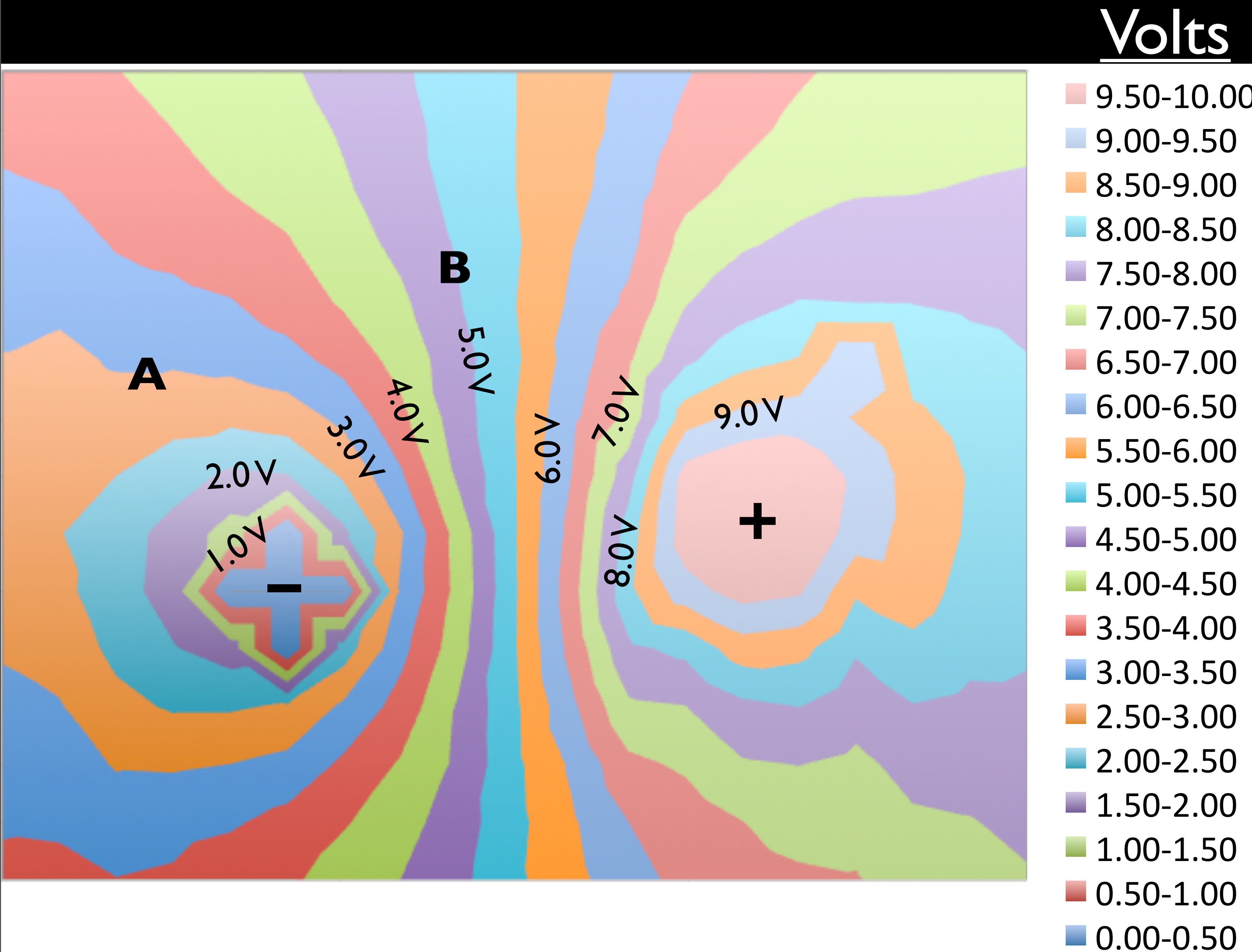
- By repeating this experiment thousands of times, Millikan discovered that the electric charge was quantized
  - charge can only have specific values, in increments of 1.6×10^{-19} C
- Millikan concluded that this must be the minimum charge possible, therefore the charge on an electron and proton
- Won Nobel Prize in Physics in 1923

Potential Mapping Lab Results



- Each color band represents a range of ΔV values
- Boundaries between bands represent locations with the same value of ΔV (*equipotentials*)

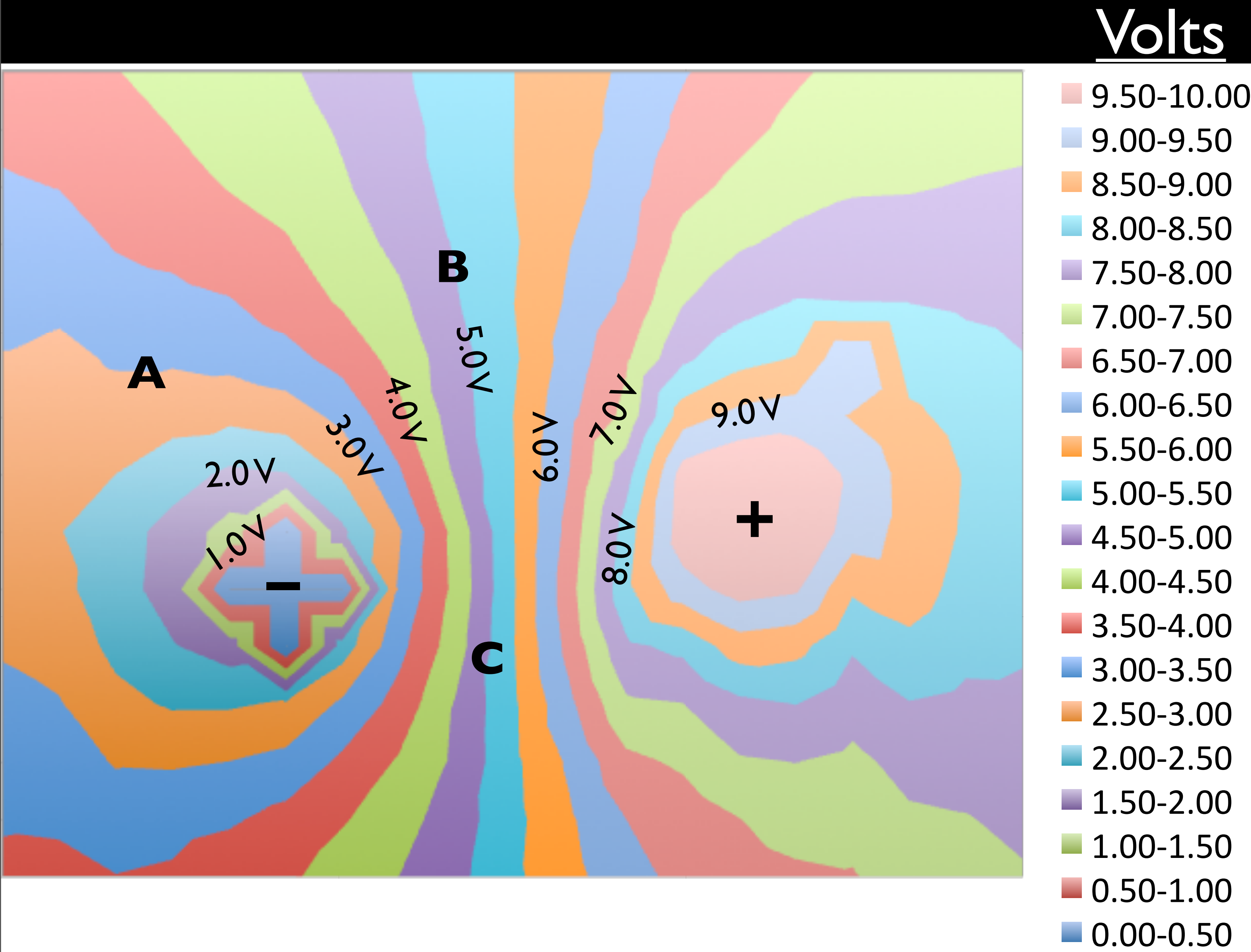
Potential Mapping Lab Results



- The work required to move a charge between two points in the electric field: $W = \Delta V q$
- How much work must you do to move a proton between points A and B?

3.2×10^{-19} J

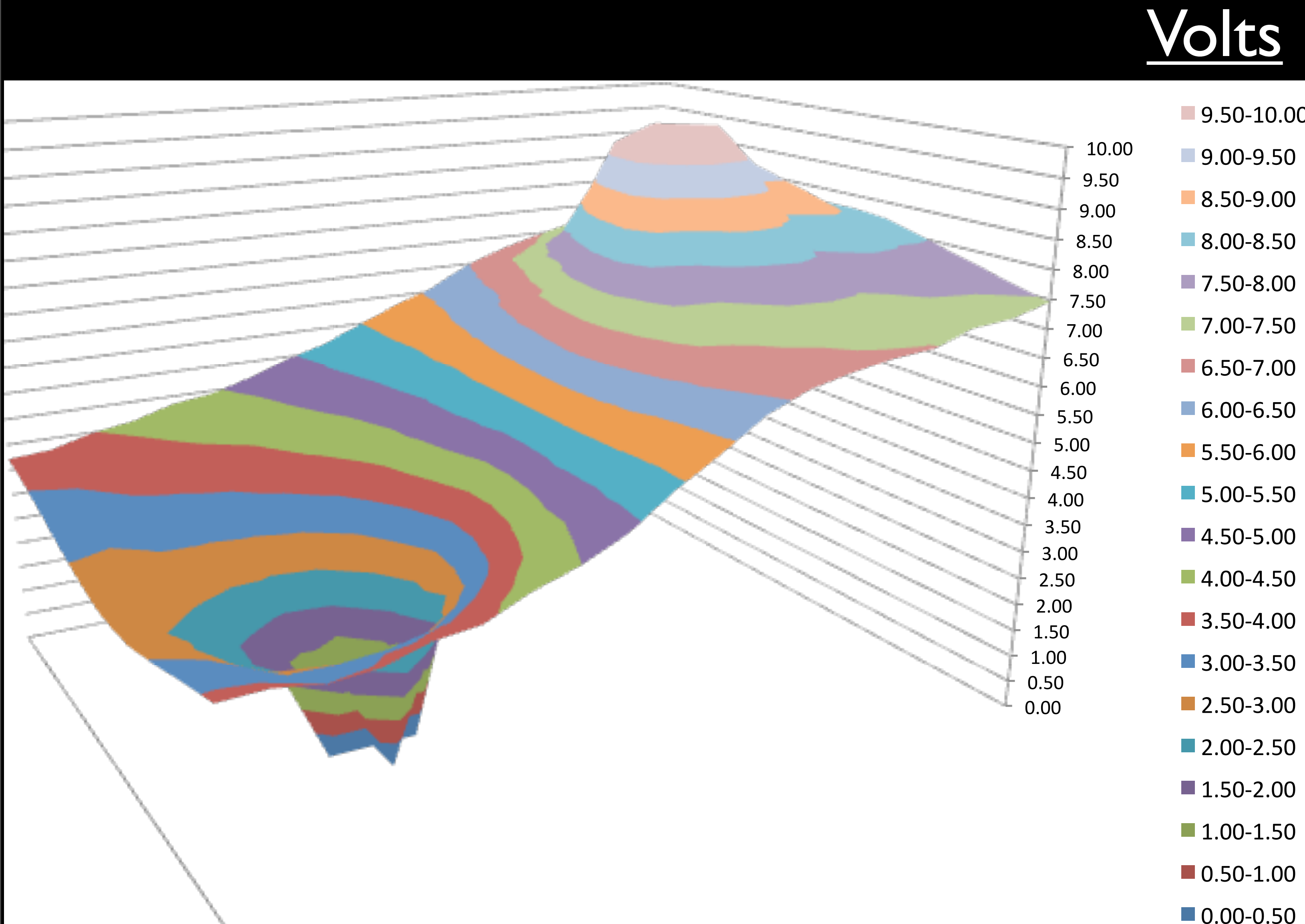
Potential Mapping Lab Results



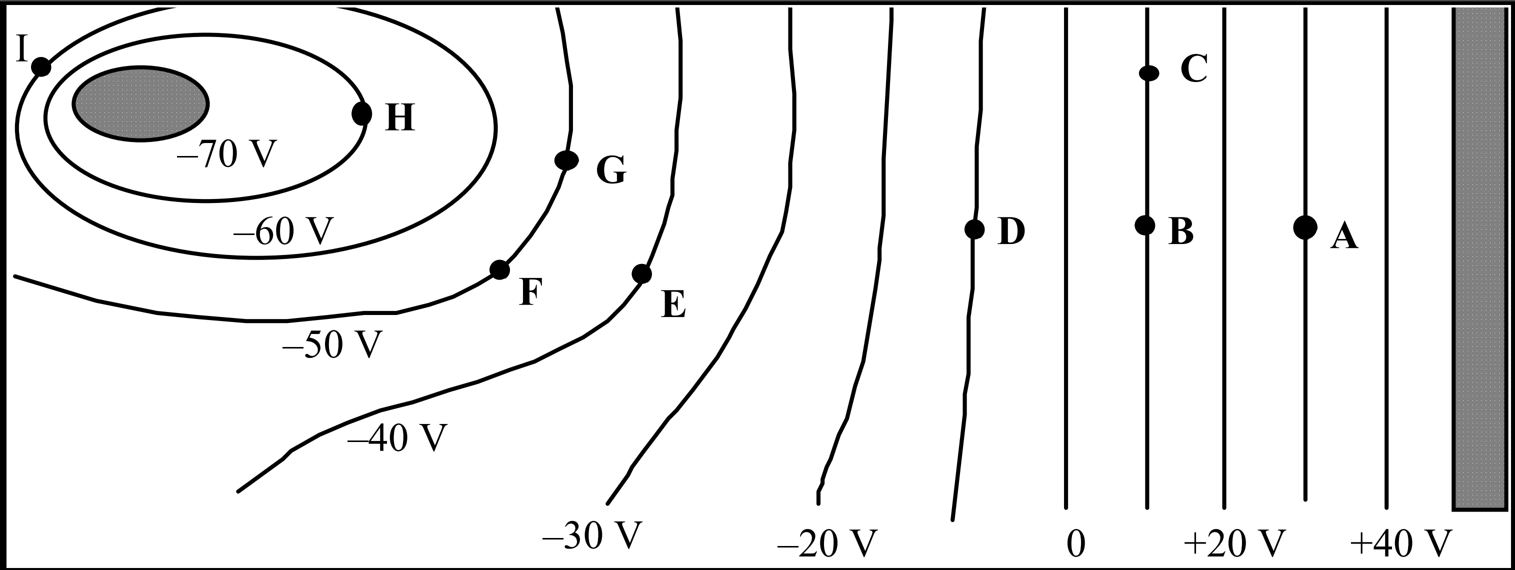
- The work required to move a charge between two points in the electric field: $W = \Delta V q$
- How much work must you do to move a proton between points B and C?

Zero J

Potential Mapping Lab Results



- A ball placed on the graph would roll down hill to lower PE...charge also moves to lower PE due to E field
- Positive charge will move from High V to Low V due to the E field (down hill)
- Negative charge will move from Low V to High V due to the E field (up hill)



- What is the potential difference between points B and E?

50 V

- How much work must you do to move a +1 μC charge from E to A?

7×10^{-5} J

- What is the direction of the electric field at B?

Towards D

(E field points in direction a positive charge would be pushed)