Unit 5: Energy

Where does energy come from and where does it go?

What is Energy?

- Energy: The ability to do work
    - Energy is a SCALAR quantity
- Two main types of energy we will be dealing with in this class: kinetic and potential
- Kinetic Energy: energy of motion
- Potential Energy: stored energy

Kinetic Energy

- An object has kinetic energy when it is in motion

Definition of Kinetic Energy

$$KE = \frac{1}{2}mv^2$$

Object A and B have the same speed, but A is twice as massive as B. How do their kinetic energies compare?

Object C and D have the same mass, but C is moving twice as fast as D. How do their kinetic energies compare?

The unit for kinetic energy is the Joule (J).

Potential Energy

- An object has gravitational potential energy due to its location in the Earth's gravitational field.

Definition of Gravitational Potential Energy

$$PE_g = mgh$$

- The height "h" is measured from an arbitrary zero level
- Heights above the zero level are positive; heights below the zero level are negative

The unit for potential energy is also the Joule (J).

- A 2-kg book sits on top of a 0.8-m tall table on the second story of a building. The floor of the second story is 4 meters above the ground. What is the potential energy of the book (a) relative to the second floor? (b) relative to the ground?

(a) $PE_g = mgh = (2\ \text{kg})(9.8\ \text{m/s}^2)(0.8\ \text{m}) = 15.68\ \text{J}$

(b) $PE_g = mgh = (2\ \text{kg})(9.8\ \text{m/s}^2)(4.8\ \text{m}) = 94.08\ \text{J}$

- The book is dropped out a window and lands on the ground. What is the potential energy of the book (a) relative to the second floor? (b) relative to the ground?

(a) $PE_g = mgh = (2\ \text{kg})(9.8\ \text{m/s}^2)(-4\ \text{m}) = -78.4\ \text{J}$

(b) $PE_g = mgh = (2\ \text{kg})(9.8\ \text{m/s}^2)(0\ \text{m}) = 0\ \text{J}$

Regardless of reference level, the change in Potential Energy between the two heights is the SAME ($\Delta PE = -94.08$ J)!

So, What is "Work"?

- Work: The transfer of energy from one object to another, or the conversion of energy from one form to another
- Work is done when a force is exerted on an object, causing a displacement

Definition of Work $$W = Fd\cos\theta$$

Units:
$N \cdot m$ = Joule (J)

F: force exerted on the object

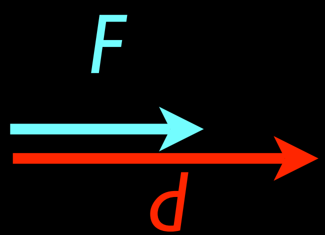
d: displacement of the object

θ: angle between force and displacement vectors

So, What is "Work"?

- What role does the angle play?

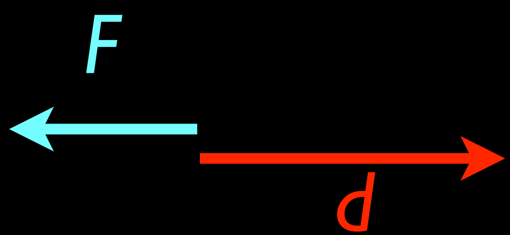
$$W = Fd\cos\theta$$



$\theta = 0$ degrees

$\cos\theta = 1$

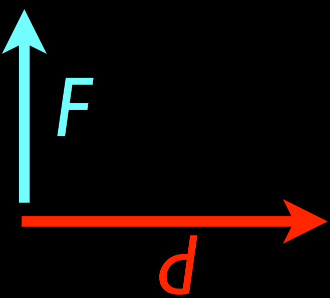
$W > 0$



$\theta = 180$ degrees

$\cos\theta = -1$

$W < 0$



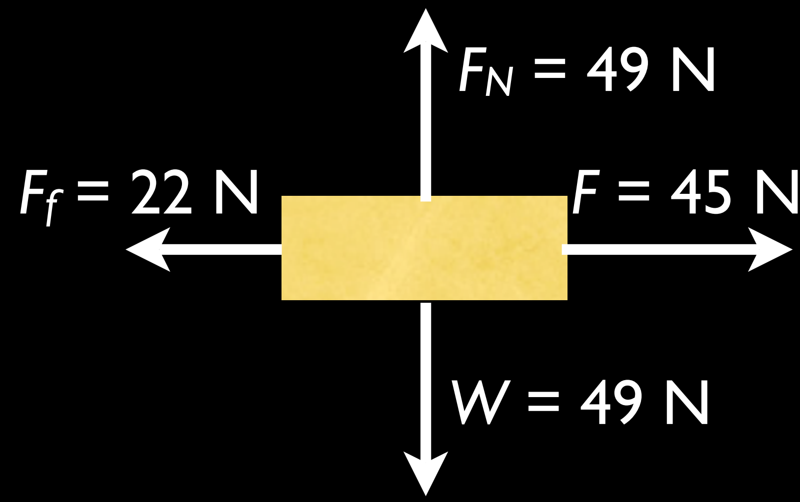
$\theta = 90$ degrees

$\cos\theta = 0$

$W = 0$

- A 5-kg book is pushed across a horizontal table with a force of 45 N to the right. Friction between the table and the book exerts a force of 22 N. The book is displaced 8 m to the right. How much total work is done on the book?

The total (or net) work is the sum of the work done by each force.



W_F = (45 N)(8 m)(1) = 360 J

W_{Ff} = (22 N)(8 m)(-1) = -176 J

W_W = (49 N)(8 m)(0) = 0 J

W_{FN} = (49 N)(8 m)(0) = 0 J

W_{NET} = 184 J

The net work can also be found by using the $\sum F$ in the work formula.

What does Work have to do with Energy?

- The Work-Energy Theorem: "The work done on an object by a net outside force is equal to the change in the object's kinetic energy."

$$W = \Delta KE$$

If Work is Positive -> KE increases

If Work is Negative -> KE decreases

- The 5-kg book from the previous example had a total of 184 J of work done on it by outside forces. Let's say it was initially moving at 4 m/s.

(a) How much kinetic energy did it start out with?

$$KE_i = \frac{1}{2}mv^2 = (0.5)(5\text{ kg})(4\text{ m/s})^2 = 40\text{ J}$$

(b) How much kinetic energy did it have after the 8-m displacement?

$$KE_f = KE_i + W = 40\text{ J} + 184\text{ J} = 224\text{ J}$$

(c) How fast was it moving after the displacement?

$$224\text{ J} = \frac{1}{2}mv_f^2 \quad \rightarrow \quad v_f = 9.47\text{ m/s}$$

- A 0.075-kg arrow, initially at rest, is fired horizontally. The bowstring exerts a net force of 65 N on the arrow over a distance of 0.90 m. The force and displacement are in the same direction. With what speed does the arrow leave the bow?

$$W_{NET} = \sum Fd \cos\theta = (65\text{ N})(0.90\text{ m})(1) \quad = 58.5\text{ J}$$

$$W = \Delta KE$$

$$58.5\text{ J} = KE_f - 0$$

$$KE_f = 58.5\text{ J}$$

$$58.5\text{ J} = \frac{1}{2}mv_f^2 \quad \rightarrow \quad v_f = 39.5\text{ m/s}$$

- A 5-kg book is released from rest from the top of a 10-m tall building. Use the work-energy theorem to determine the speed of the book the moment before it hits the ground.

$\sum F = 49 \text{ N}$

$W_{NET} = \sum Fd \cos \theta = (49 \text{ N})(10 \text{ m})(1) \quad = 490 \text{ J}$

$W_{NET} = \Delta KE$

$490 \text{ J} = KE_f - 0$

$490 \text{ J} = \frac{1}{2} m v_f^2 \quad \rightarrow \quad v_f = 14.0 \text{ m/s}$

Conservation of Energy

- "Energy cannot be created or destroyed, but it can be converted from one form to another."
- In the absence of non-conservative forces (such as friction), the total mechanical energy in a system remains constant.
  - The glider's PE was converted into KE.

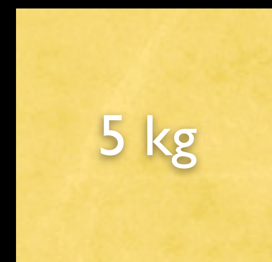
Conservation of Energy

Energy Before = Energy After

$$KE_i + PE_i = KE_f + PE_f$$

Using Conservation of Energy

- A 5-kg block is dropped from a height of 10 m. How fast is it going right before hitting the ground?



h = 10 m

Energy before = Energy after

$$\cancel{KE_i} + PE_i = KE_f + \cancel{PE_f}$$

$$(mgh)_i = (\frac{1}{2}mv^2)_f$$

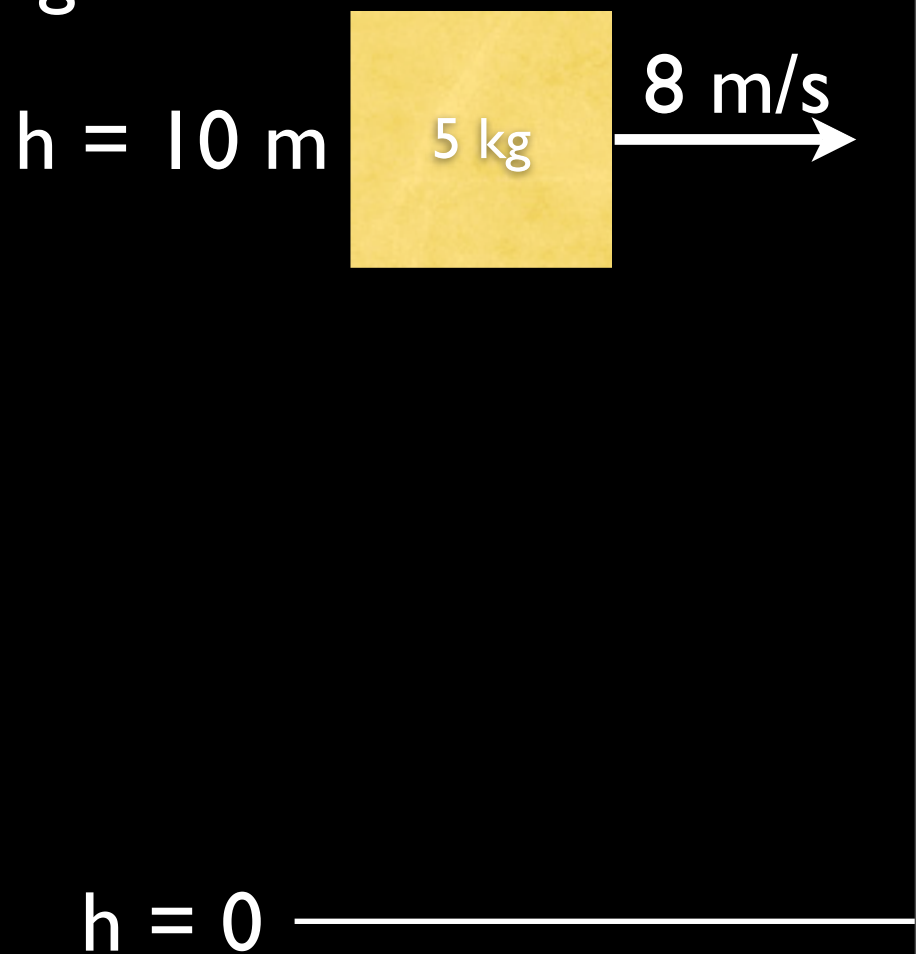
$$490\text{ J} = (\frac{1}{2}mv^2)_f$$

$$14\text{ m/s} = v_f$$

h = 0

Using Conservation of Energy

- A 5-kg block is thrown horizontally from a height of 10 m. Its initial speed is 8 m/s. How fast is it going right before hitting the ground?



Energy before = Energy after

$$KE_i + PE_i = KE_f + \cancel{PE_f}$$

$$(\frac{1}{2}mv^2)_i + (mgh)_i = (\frac{1}{2}mv^2)_f$$

$$160\text{ J} + 490\text{ J} = (\frac{1}{2}mv^2)_f$$

$$16.1\text{ m/s} = v_f$$

Using Conservation of Energy

- A 5-kg block starts at rest at the top of a frictionless inclined plane. The plane has a length of 10 m and is inclined at 30 degrees. How fast is the block moving when it reaches the bottom of the ramp?

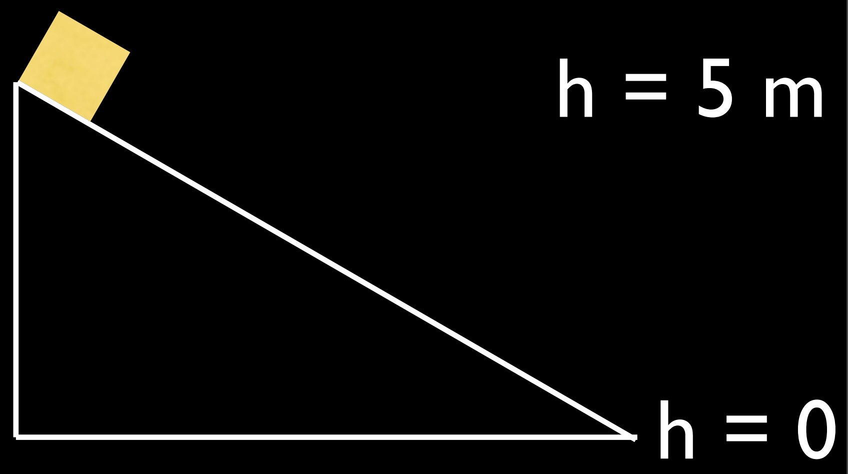
Energy before = Energy after

$$\cancel{KE_i} + PE_i = KE_f + \cancel{PE_f}$$

$$(mgh)_i = (\frac{1}{2}mv^2)_f$$

$$245\text{ J} = (\frac{1}{2}mv^2)_f$$

$$9.90\text{ m/s} = v_f$$



- One of the tallest and fastest roller coasters in the world is the Steel Dragon in Mie, Japan. The ride includes a vertical drop of 93.5 m. The coaster has a speed of 3.0 m/s at the top of the drop. Assume friction is negligible and find the speed of the riders at the bottom.

Energy before = Energy after

$$KE_i + PE_i = KE_f + \cancel{PE_f}$$

$$(\frac{1}{2}\cancel{m}v^2)_i + (\cancel{m}gh)_i = (\frac{1}{2}\cancel{m}v^2)_f$$

Mass cancels out!

$$\sqrt{(v_i^2 + 2gh_i)} = v_f$$

$$42.9 \text{ m/s} = v_f$$

- What if two students each carried a 25-kg stack of books up the 4-m displacement, but one student ran (taking 4 seconds) and the other walked (taking 10 seconds).

Definition of Power

$$P = \frac{W}{\Delta t}$$

Unit: J/s = Watt (W)

Runner -> did 980 J in 4 seconds

P_{runner} = (980 J)/(4 s) = 245 W

Walker -> did 980 J in 10 seconds

P_{walker} = (980 J)/(10 s) = 98 W