Unit 6: Momentum

What is momentum?

- Momentum is "matter in motion"

Definition of Momentum

$$\vec{p} = m\vec{v}$$

- Momentum is a vector quantity
- Units -> kg m/s

- A baseball (m = 0.14 kg) has an initial velocity of -38 m/s as it approaches a bat. After being hit by the batter, the ball has a final velocity of +58 m/s. Determine the baseball's (a) the initial momentum, (b) final momentum, and (c) change in momentum.

(a) $p = mv$ = (0.14 kg)(-38 m/s) = -5.32 kg m/s

(b) $p = mv$ = (0.14 kg)(+58 m/s) = +8.12 kg m/s

(c) $\Delta p = p_f - p_i$ = (+8.12 kg m/s) - (-5.32 kg m/s)

= +13.4 kg m/s

- Newton's 2nd Law was originally written in terms of momentum:
  - $\sum F = \Delta p / \Delta t$

Definition of Impulse

$$\vec{J} = \vec{F}\,\Delta t$$

Units: N · s

Impulse-Momentum Theorem

$$\vec{J} = \Delta \vec{p}$$
$$\vec{F}\,\Delta t = m\,\Delta \vec{v}$$

- The baseball from the previous problem experienced a change in momentum of +13.4 kg m/s. If the bat was in contact with the ball for 0.0016 s, how much force (on average) did it exert during this time period?

J = +13.4 kg m/s $= F\Delta t$

F = (+13.4 kg m/s)/(0.0016 s)

= +8400 N

- A 240-kg snowmobile accelerates from +6 m/s to +28 m/s.
  (a) Determine the snowmobile's initial momentum and final momentum.
  (b) What impulse does the snowmobile experience?
  (c) If the acceleration took 60 s, what average force acted on the snowmobile?
  (d) If the acceleration took 30 s, what average force acted on the snowmobile?

(a) p_i = (240 kg)(+6 m/s) = +1440 kg m/s
p_f = (240 kg)(+28 m/s) = +6720 kg m/s

(b) $J = \Delta p$ = 6720 kg m/s - 1440 kg m/s = +5280 N · s

(c) F = (+5280 N · s)/(60 s) = +88 N

(d) F = (+5280 N · s)/(30 s) = +176 N

- A hockey player strikes a stationary puck (m = 0.5 kg) with an average force of +250 N. Determine the impulse delivered and final velocity of the puck if the force is applied for: (a) 0.05 s, and (b) 0.10 s.

(a) $J = F\Delta t = (+250\ \text{N})(0.05\ \text{s}) = +12.5\ \text{N}\cdot\text{s} = p_f - p_i$

$v_f = (+12.5\ \text{N}\cdot\text{s})/(0.5\ \text{kg}) = +25\ \text{m/s}$

(b) $J = F\Delta t = (+250\ \text{N})(0.10\ \text{s}) = +25\ \text{N}\cdot\text{s} = p_f - p_i$

$v_f = (+25\ \text{N}\cdot\text{s})/(0.5\ \text{kg}) = +50\ \text{m/s}$

- A driver crashes head on into a tree at +25 m/s. Without an airbag, the 80-kg driver is brought to rest in 0.05 s. With an airbag, the driver is brought to rest in 0.4 s. Determine the impulse acting on the driver in each instance and the force required to stop him in each case.

(a) $J = \Delta p = p_f - p_i = 0 - (80\text{ kg})(+25\text{ m/s}) = -2000\text{ N} \cdot \text{s}$

(b) Same impulse (change in momentum) required

(a) $F = J/\Delta t = (-2000\text{ N} \cdot \text{s})/0.05\text{ s} = -40{,}000\text{ N}$

(b) $F = J/\Delta t = (-2000\text{ N} \cdot \text{s})/0.4\text{ s} = -5{,}000\text{ N}$

The airbag reduced the force exerted on the driver by 87.5%!

Conservation of Momentum

- In any collision or explosion, the total momentum of all objects involved is conserved.
  - To prove this we start with Newton's Third Law

Each object feels this force during the same time interval

$$-F_{B \text{ on } A} = F_{A \text{ on } B}$$

$$-F_{B \text{ on } A}\,\Delta t = F_{A \text{ on } B}\,\Delta t$$

$$-\Delta p_A = \Delta p_B$$

$$-(p_{Af} - p_{Ai}) = p_{Bf} - p_{Bi}$$

$$-p_{Af} + p_{Ai} = p_{Bf} - p_{Bi}$$

$$p_{Ai} + p_{Bi} = p_{Af} + p_{Bf}$$

$\text{Momentum}_{\text{BEFORE}} = \text{Momentum}_{\text{AFTER}}$

Conceptual Examples

- Two ice skaters, initially at rest standing on a frictionless surface, push off of each other.
  - What is the total momentum before they push off?

    Zero!
  - What must the total momentum be after they push off?

    Zero!
  - What does this tell us about the the individual momentum of each skater?

    Same magnitude, opposite direction

    More massive skater will have lower speed

Conceptual Examples

- Object A with a mass of m traveling at speed of v_1 collides with object B (also with a mass of m) initially at rest. They stick together after the collision.
  - What is the total momentum before the collision? $p_i = mv_1$
  - What must the total momentum be after the collision? $p_f = mv_1 = (2m)v_2$
  - Will they be moving faster than v, slower than v, or with a speed of v after the collision?

Speed must be less because mass is more

Since mass is exactly doubled, speed is exactly half!

Collision Example

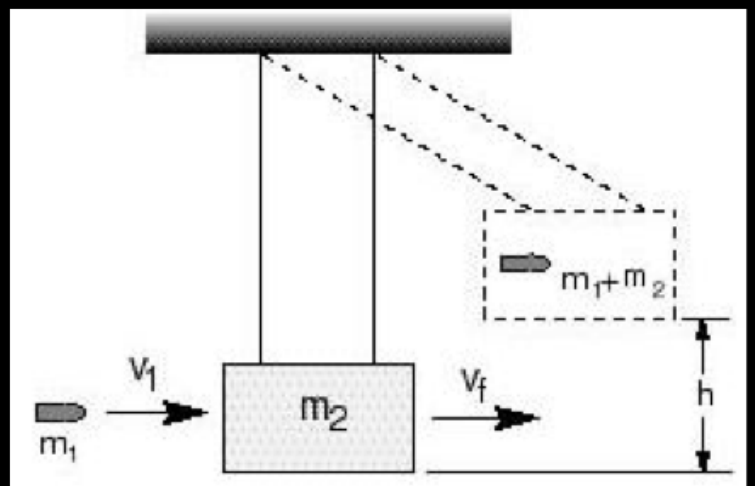
- A 2500-kg SUV traveling at 32 m/s strikes a stationary car (mass = 1100 kg). The two vehicle become interlocked. Determine the velocity the wreckage travels at immediately following the collision.

Explosion Example

- A 5-kg rifle fires a 4.2-g bullet at 956 m/s. Determine the recoil velocity of the rifle.

Ballistic Pendulum

- A 7.5-g bullet is fired with an initial velocity of 375 m/s into a 2.4-kg suspended block (as shown). The bullet becomes embedded in the block, and the combination swings upward to a maximum height of *h*.
  (a) Determine the velocity of the block/bullet after the collision.
  (b) Determine the maximum height of the pendulum.



(a) Conservation of Momentum:

$$p_{\text{bullet}i} = p_{\text{combo}f}$$

$$(0.0075\ \text{kg})(375\ \text{m/s}) = (2.4075\ \text{kg})v_f$$

$$1.17\ \text{m/s} = v_f$$

Ballistic Pendulum

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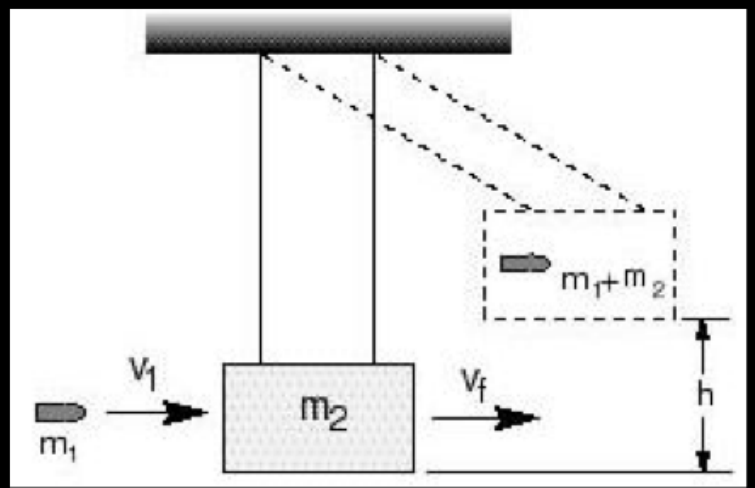


(b) Conservation of Mechanical Energy

$$KE_i + PE_i = KE_f + PE_f$$

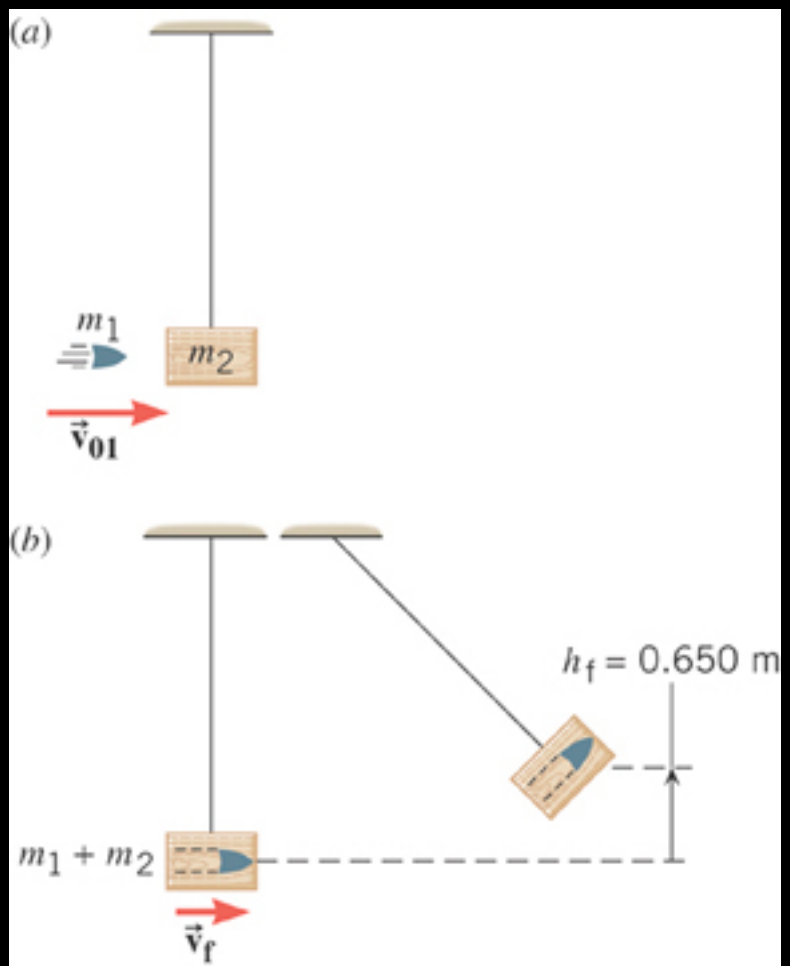
$$\left(\frac{1}{2}mv^2\right)_i = (mgh)_f$$

$$1.65\ \text{J} = (mgh)_f$$

$$0.0699\ \text{m} = h_f$$

Ballistic Pendulum

- A 10.0-g bullet is fired into the 2.50-kg stationary block of a ballistic pendulum, and the block/ bullet swing to a maximum height of 0.650 m above the initial position. Find the speed with which the bullet is fired.



v = 896 m/s