Unit 4:
Uniform Circular Motion

How do Newton's Laws apply to circular motion?

- Name 3 driver-controlled devices that can directly cause a car to accelerate.
    - Gas pedal - causes an increase in the magnitude of velocity.
    - Brake pedal - causes a decrease in the magnitude of velocity.
    - Steering wheel - causes a change in the direction of the velocity.

Uniform Circular Motion

- U.C.M. → An object traveling at constant speed in a circular path
- How can we determine the speed (magnitude of velocity) of an object traveling in a circle?
  - Distance = Circumference of circle ($2\pi r$)
  - Time = Period for one revolution (T)

U.C.M. Speed

$$v = \frac{2\pi r}{T}$$

- A toy car traveling on a circular path (diameter = 90 cm) completes a lap every 1.5 s. Determine the speed of the car.

$r = 0.45$ m
$T = 1.5$ s

$$v = \frac{2\pi r}{T}$$

$$= \frac{2\pi(0.45 \text{ m})}{1.5 \text{ s}}$$

$$= 1.88 \text{ m/s}$$

- The wheel of a car has a radius of 0.29 m and is rotating at 830 revolutions per minute (rpm). Determine the speed (in m/s) at which the outer edge of the wheel is moving.

$$830\ \frac{\text{revolutions}}{\text{min}} \times \frac{1\ \text{min}}{60\ \text{s}} = 13.8\ \text{rev/s} \rightarrow 0.0723\ \text{s/rev}$$

$T = 0.0723$ s
$r = 0.29$ m

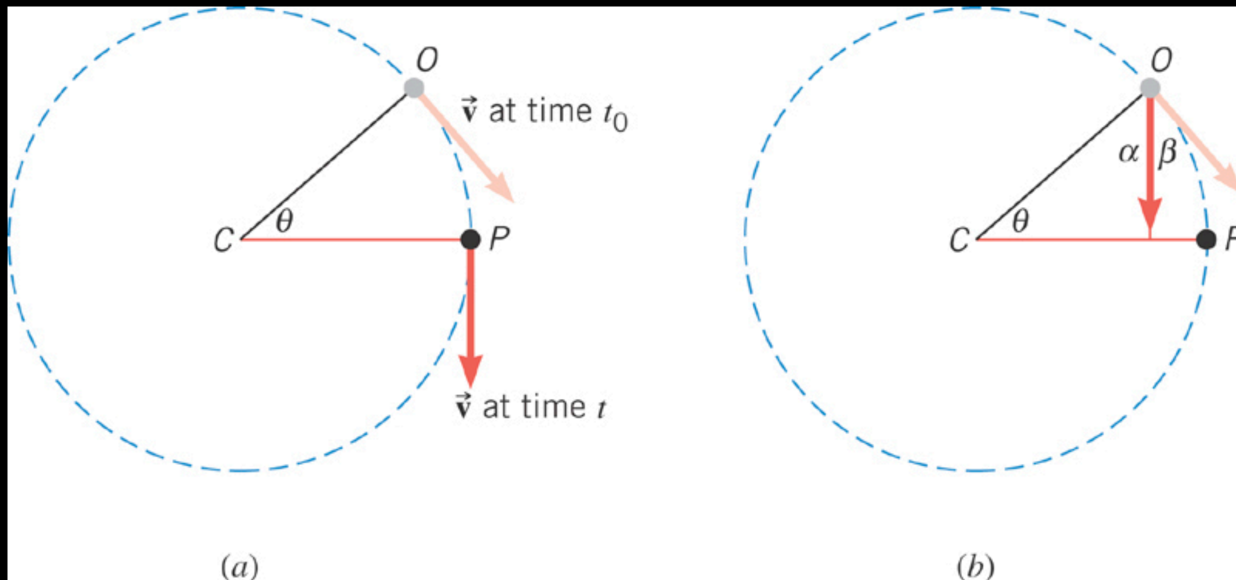
$$v = \frac{2\pi r}{T}$$

$$= \frac{2\pi(0.29\ \text{m})}{0.0723\ \text{s}}$$

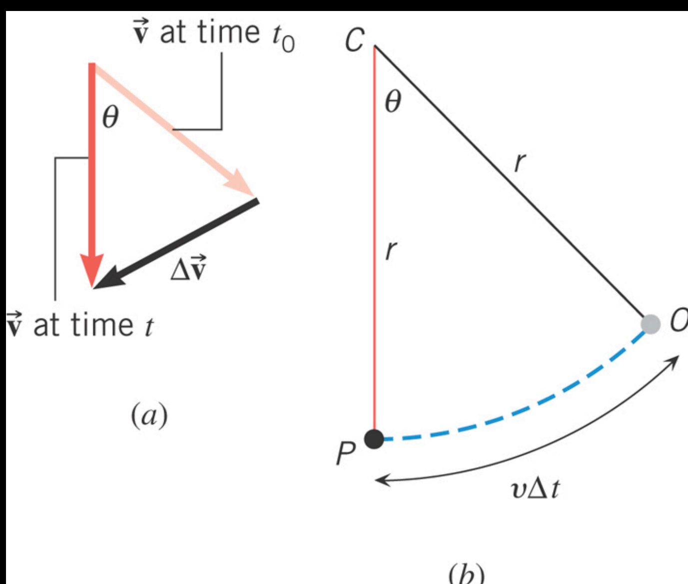
$$= 25.2\ \text{m/s}$$

- An object in U.C.M. has constant speed, but the velocity is NOT constant (changing direction)
- The object is experiencing a centripetal acceleration pointing towards the center of the circular path
    - Centripetal = “center-seeking”

Derivation of Centripetal Acceleration (a_c)



$\beta = \theta$

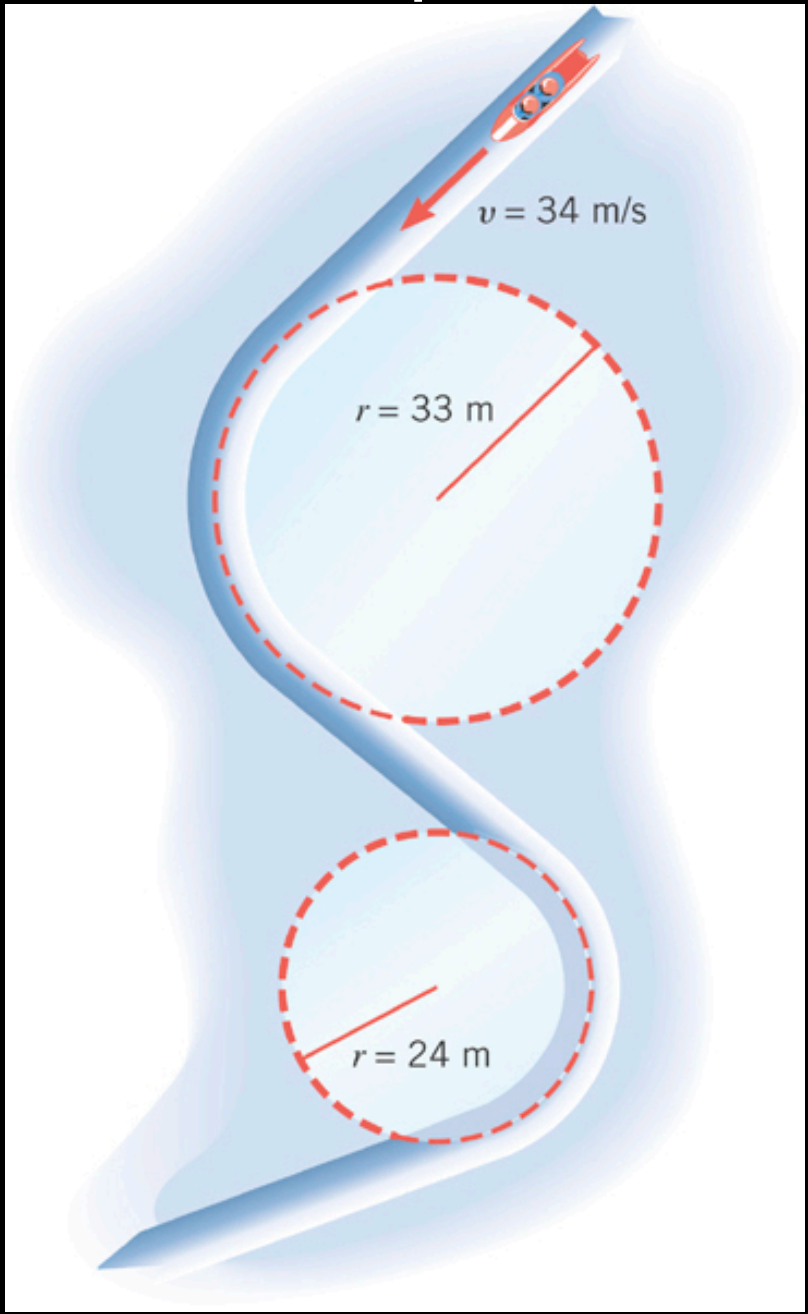


Similar Isosceles Triangles: $\dfrac{\Delta v}{v} = \dfrac{v\Delta t}{r}$

Centripetal Acceleration

$$a_c = \frac{v^2}{r}$$

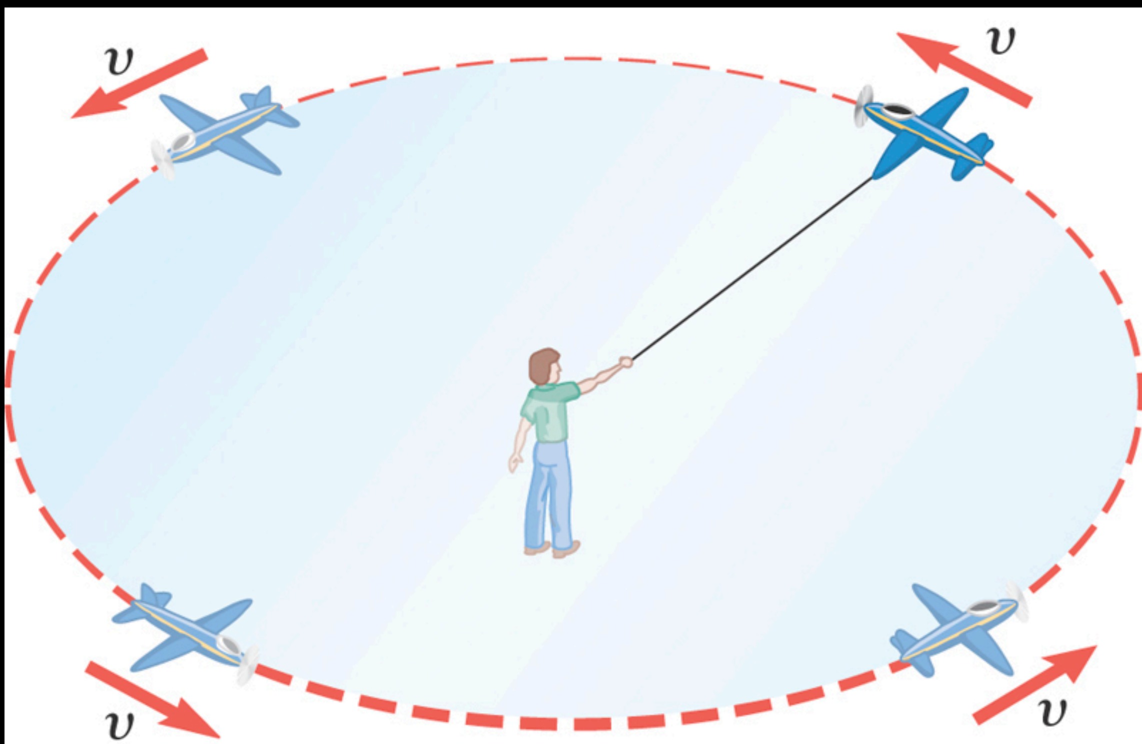
- Determine the centripetal acceleration the bobsled will experience in each turn illustrated.



$$a_c = \frac{(34\ \text{m/s})^2}{33\ \text{m}} = 35.0\ \text{m/s}^2$$

$$a_c = \frac{(34\ \text{m/s})^2}{24\ \text{m}} = 48.2\ \text{m/s}^2$$

- A 0.90-kg toy airplane travels in a uniform circular path due to a 17-m guidewire. Determine the centripetal acceleration the plane experiences when traveling with a speed of (a) 19 m/s, and (b) 38 m/s.



(a) $a_c = \dfrac{(19\ \text{m/s})^2}{17\ \text{m}}$

$= 21.2\ \text{m/s}^2$

(b) $a_c = \dfrac{(38\ \text{m/s})^2}{17\ \text{m}}$

$= 84.9\ \text{m/s}^2$

Doubling the speed quadruples the required centripetal acceleration.

Do Now:

- Car A negotiates a curve at a speed of 32 m/s and experiences a centripetal acceleration of 6.4 m/s^2. Car B negotiates the same curve at a speed of 16 m/s. What centripetal acceleration does it experience?

Conceptually:

$$a_c \propto v^2$$

"Proportional to"

If velocity is cut in half, the acceleration will be $(1/2)^2$ as large

$$a_{c\text{CAR B}} = (1/2)^2 \times a_{c\text{CAR A}}$$

$$= 1.6\ m/s^2$$

Centripetal Force

- When an object travels in a circle, it is accelerating...therefore it has a net force acting on it ($\sum F \neq 0$)
- When a net force causes circular motion, it is called the centripetal force (F_c).
- IMPORTANT: Centripetal force is NOT a new category of force. It is simply what we call the net force causing the circular motion. (It could be a single force or the vector sum of multiple forces.)

Examples of F_c

- A toy plane flying in a horizontal circle at the end of a string.

  Tension is F_c

- A marble rolling around the inside of an upside-down wine glass.

  Normal Force is F_c

- The Moon traveling in a (nearly) circular orbit around the Earth.

  Force of Gravity is F_c

- A car driving around a horizontal turn without skidding.

  Static Friction is F_c

F_c and Newton's 2nd Law

- Newton's 2nd Law: $\sum F = ma$

- Since $\sum F$ is the centripetal force: $F_c = ma$

- The acceleration experienced is a_c: $a = v^2/r$

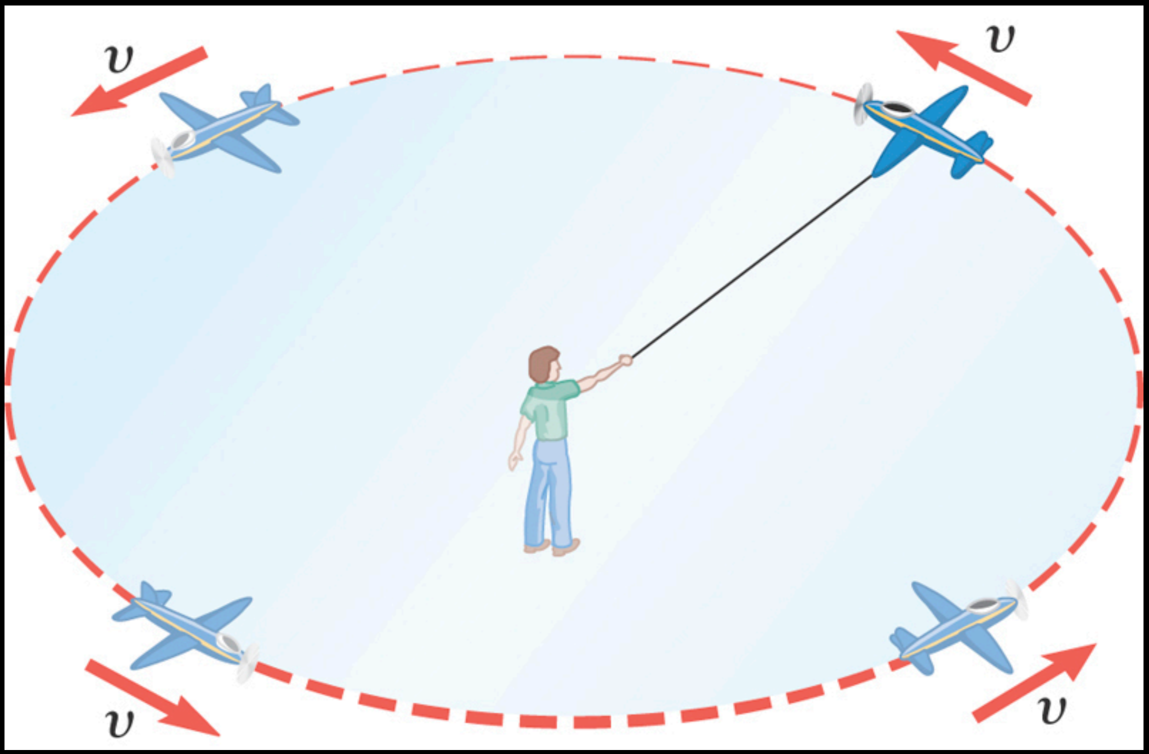
Magnitude of (net) Centripetal Force:

$$F_C = m\frac{v^2}{r}$$

Direction: Towards center of circle

Toy Plane: Revisited

- A 0.90-kg toy airplane travels in a uniform circular path due to a 17-m guidewire. Determine the tension in the wire (i.e. centripetal force) when the plane is traveling with a speed of (a) 19 m/s, and (b) 38 m/s.



(a) $F_c = 19.1$ N

(b) $F_c = 76.4$ N

Doubling the speed quadruples the centripetal force required.

Driving Around a Turn

- A driver wishes to make a left turn while maintaining a speed of 18.0 m/s. The path of his turn is part of a circle of radius 15.0 m. What centripetal acceleration will the car experience?

$$21.6\ \text{m/s}^2$$

- What is the minimum coefficient of static friction between the tires and the road required for this maneuver?

$$F_{f\,MAX} = ma_c$$

$$\mu_s F_N = ma_c$$

$$\mu_s(mg) = ma_c$$

$$\mu_s = 2.20$$

Newton's Biggest Idea: Universal Gravity

- Any object with mass exerts an attractive force on any other object with mass, which we call gravity
- Newton determined that this force must depend on each object's mass, and on the distance between them

Force of Universal Gravity

$$F_g = \frac{Gm_1m_2}{r^2}$$

- *G* is called the universal gravitation constant
- Measured by Henry Cavendish (1798)
- $G = 6.67 \times 10^{-11}\ \mathrm{N{\bullet}m^2/kg^2}$

$$F_g = \frac{Gm_1m_2}{r^2}$$

- Two spheres have their centers 2.0 m apart. One sphere has a mass of 8.0 kg; the other a mass of 6.0 kg. What is the gravitational force between them? 8.00×10^{-10} N
- Two bowling balls each have a mass of 6.8 kg. Their centers are separated by 21.8 cm. Find the gravitational force they exert on each other. 6.49×10^{-8} N
- Determine the gravitational force the Earth (m = 5.98×10^{24} kg) exerts on the Moon (m = 7.36×10^{22} kg), if they have an average separation distance of 3.85×10^{8} m.

1.98×10^{20} N

$$F_g = \frac{Gm_1m_2}{r^2}$$

- Two objects of mass m_1 and m_2, respectively, are separated by a distance r. They exert a force of F on each other. What happens to the magnitude of the force if:
  - mass m_1 is doubled? x2
  - mass m_1 and m_2 are doubled? x4
  - the distance r is doubled? ÷4
  - the distance r and both masses are doubled? Same

Universal Gravity & Weight

Tying it all together...

- How does this law of universal gravity relate to our concept of "weight"?

$$W = F_g$$

$$m_1 g = \frac{G m_1 m_{EARTH}}{r_{EARTH}^2}$$

$$g = \frac{G m_{EARTH}}{r_{EARTH}^2}$$

Acceleration Due to Gravity on the Surface of a Planet

$$g = \frac{G m_{PLANET}}{r_{PLANET}^2}$$

- The planet Mars has a radius of 3.40×10^6 m and a mass of 6.42×10^{23} kg. Determine the acceleration due to gravity an object in freefall would experience near its surface.

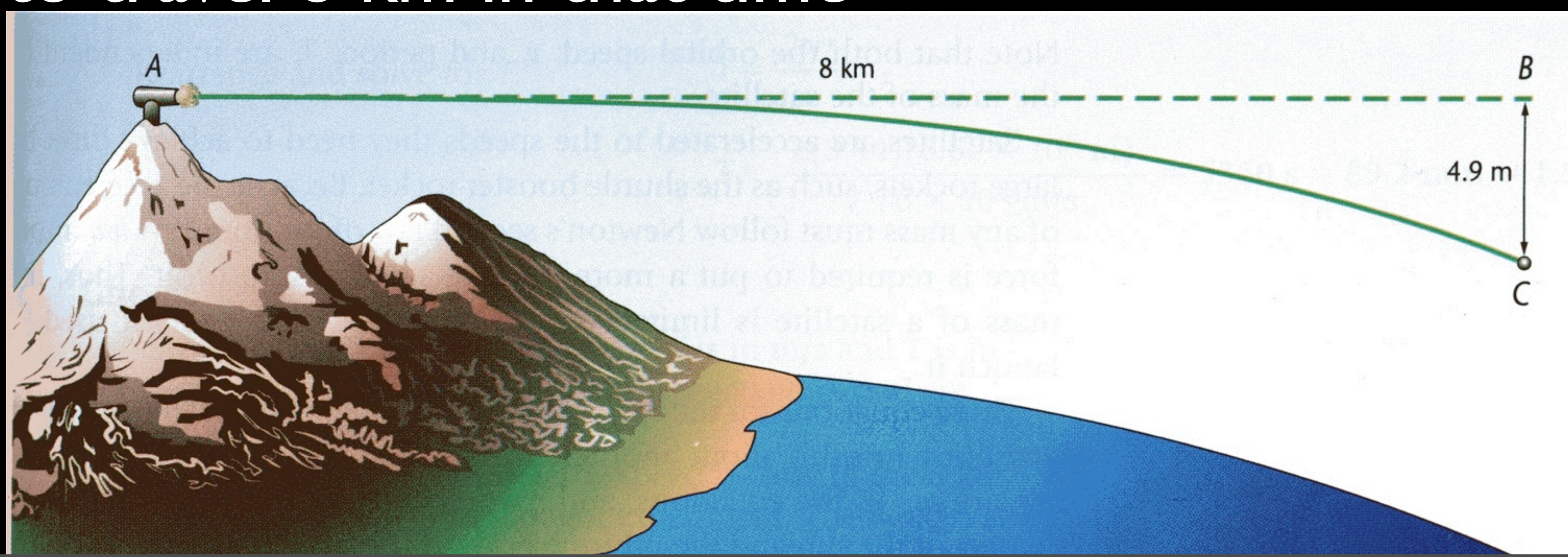
$$g = 3.70 \text{ m/s}^2$$

- The planet Venus has a radius of 6.05×10^6 m and a mass of 4.87×10^{24} kg. Determine the acceleration due to gravity an object in freefall would experience near its surface.

$$g = 8.87 \text{ m/s}^2$$

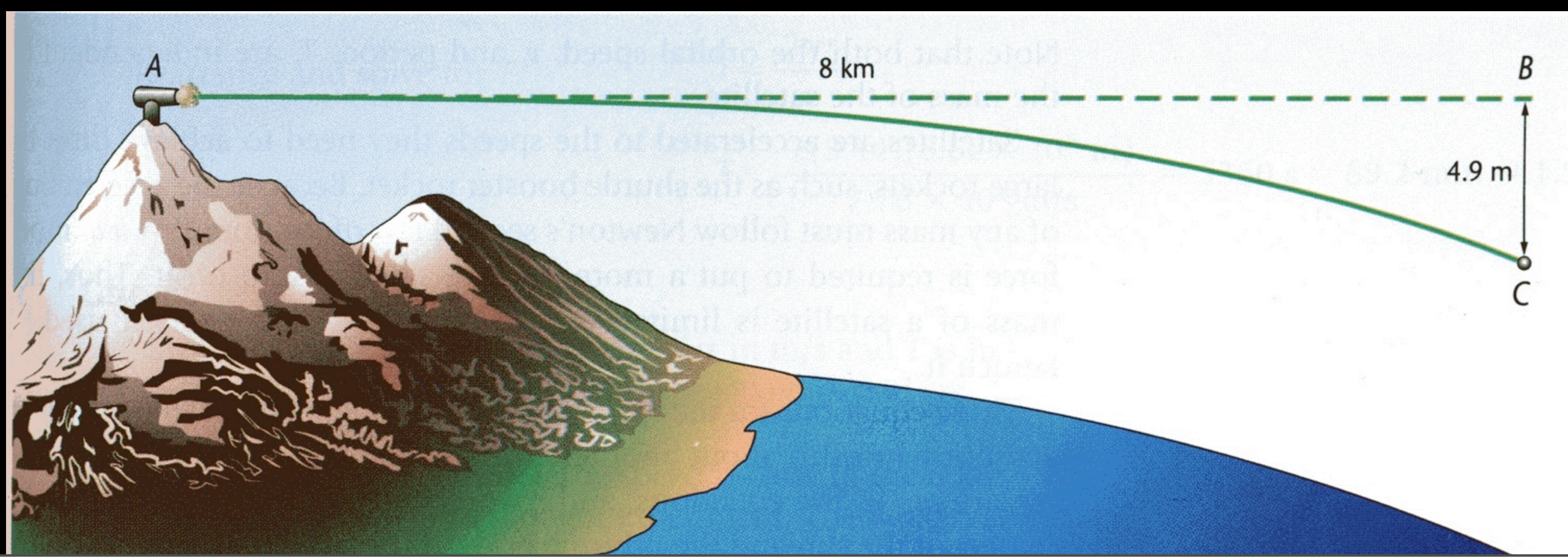
Satellites in Circular Orbits

- Imagine you fired a cannon from the top of a mountain...
  - Cannonball falls a vertical distance of 4.9 m in the first second
  - To match curvature of Earth, cannonball would need to travel 8 km in that time



Satellites in Circular Orbits

- Near the surface of the Earth, the velocity required to orbit the Earth is 8 km/s
  - What about at higher altitudes?



Satellites in Circular Orbits

- Centripetal force = force of gravity

$$\sum F = ma$$

$$F_c = ma_c$$

$$\frac{Gm_{satellite}m_{planet}}{r^2} = m_{satellite}\frac{v^2}{r}$$

"r" is the radius of the satellite's orbit

Speed of a Satellite in Circular Orbit

$$v_{satellite} = \sqrt{\frac{Gm_{planet}}{r}}$$

- A satellite orbits the Earth in a circular orbit of radius 6.61×10^6 m. (a) What is the satellite's speed? (b) How much time does it take to orbit the Earth?

v = 7770 m/s T = 5350 s (1.5 hours)

- Determine the speed of the Hubble Space Telescope, which orbits at a height of 598 km above the Earth's surface (radius of the Earth = 6.38×10^6 m).

v = 7560 m/s

- Astronomers examining galaxy M87 have found that the orbiting speed of matter located a distance of 5.7×10^{17} m from the galaxy center is 7.5×10^5 m/s. Determine the mass of the object located at the galactic center. Compare this to the mass of the Sun (2×10^{30} kg).

$$M = 4.8 \times 10^{39} \text{ kg} \qquad = 2.4 \text{ Billion Suns!}$$

- Some satellites travel in geosynchronous orbit, meaning they take exactly one day to go around the Earth (appears motionless from the surface). At what height above the Earth's surface do these satellites orbit?

$$r_{orbit} = 4.32 \times 10^7 \text{ m}$$

$$h = r_{orbit} - r_{earth} = 3.59 \times 10^7 \text{ m}$$