Lecture Notes: (5.4) Integration of Exponential Functions
Marshall Math

Integrals for Exponential Functions

Theorem: Integration Rules for Exponential Functions

Let u be a differentiable function of x.

1. $\int e^x dx = e^x + C$ 2. $\int e^u du = e^u + C$

Example: Using the Log Rule for Integration

Evaluate $\int e^{3x+1} dx$

Solution: Let $u = 3x+1$, then $du = 3dx$

$$= \frac{1}{3}\int e^{3x+1}(3)dx = \frac{1}{3}\int e^u du$$

$$= \frac{1}{3}e^u + C$$

$$= \frac{1}{3}e^{3x+1} + C$$

Example: Using the Log Rule for Integration

Evaluate ~~$\int 5e^{x^2} dx$~~ $\int 5x e^{-x^2} dx$

Solution: Let $u = -x^2$, then $du = -2x\,dx$ OR $x\,dx = -\frac{1}{2}du$

$$= 5\int e^{-x^2}(x\,dx) = 5\int e^u\left(-\frac{1}{2}du\right)$$

$$= -\frac{5}{2}\int e^u du$$

$$= -\frac{5}{2}e^u + C = -\frac{5}{2}e^{-x^2} + C$$

Example:

a. $\int \frac{e^{1/x}}{x^2}dx = -\int e^{1/x}\left(-\frac{1}{x^2}\right)dx$

$= -\int e^u du = -e^{1/x} + C$

Let $u = \frac{1}{x} = x^{-1}$

$du = -1x^{-2} = -\frac{1}{x^2}$

b. $\int \sin x e^{\cos x}dx$

$= -\int e^{\cos x}(-\sin x\,dx) = -e^{\cos x} + C$

Let $u = \cos x$

$du = -\sin x\,dx$

Example:

a. $\int_0^1 e^{-x}dx = -e^{-x}\Big]_0^1 = -e^{-1} - (-1) = 1 - \frac{1}{e} \approx 0.632$

b. $\int_0^1 \frac{e^x}{1+e^x}dx = \ln(1+e^x)\Big]_0^1 = \ln(1+e) - \ln 2$

≈ 0.620

c. $\int_{-1}^0 \left[e^x \cos\left(e^x\right)\right]dx = \sin(e^x)\Big]_{-1}^0$

$= \sin(e^0) - \sin(e^{-1})$

$= \sin 1 - \sin(e^{-1}) \approx 0.482$

HW p. ~~344~~ 346, #75-85:odd, #89