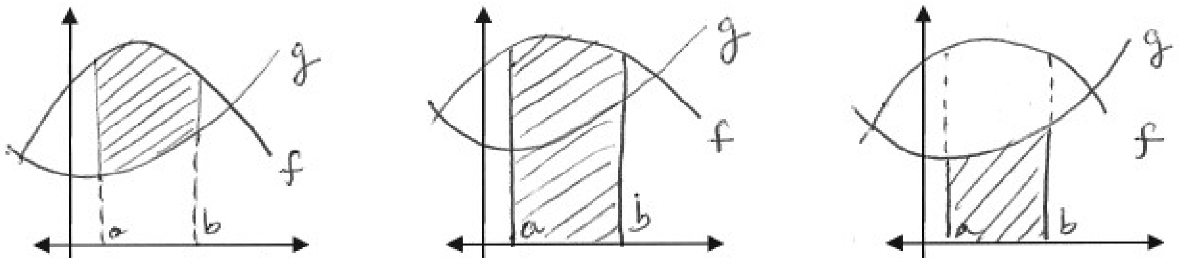
Area of a Region Between Two Curves

With a few modifications you can extend the application of definite integrals from area UNDER the curve to the area BETWEEN two curves.



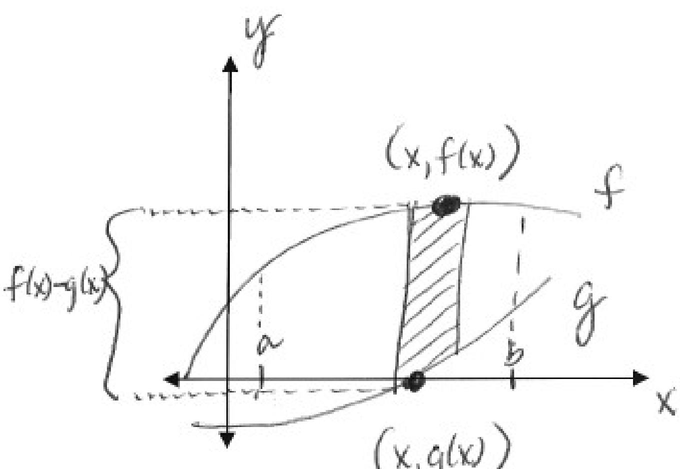
Area b/w f & g = Area under f − Area under g

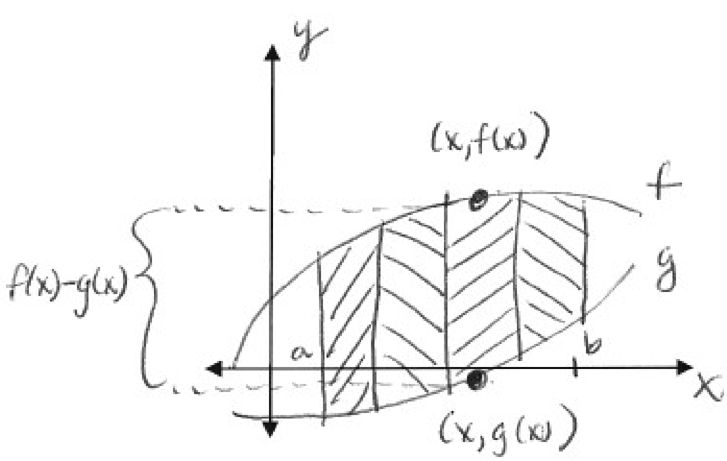
$$\int_a^b [f(x)-g(x)]\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

Area of a Region Between Two Curves

If a function f and g are continuous on $[a,b]$ and $g(x) \le f(x)$ for all x in $[a,b]$, then the area of the region bounded by the graphs of f and g and the vertical lines $x = a$ and $x = b$ is

$$A = \int_a^b \left[f(x) - g(x) \right] dx$$





Example:

Find the area of the region bounded by the graphs of $y = x^2 + 2$, $y = -x$, $x = 0$, and $x = 1$

<u>Solution:</u> Let $f(x) = x^2+2$
$g(x) = -x$ Then $g(x) \le f(x)$ for all x in $[0,1]$

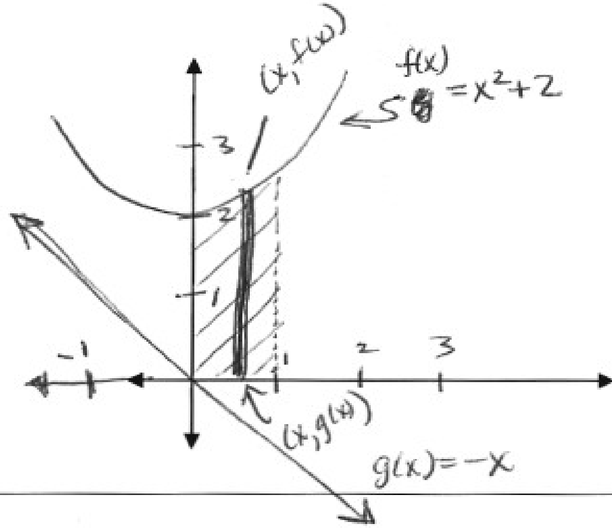
$$A = \int_a^b [f(x) - g(x)]\,dx$$

$$= \int_0^1 [(x^2+2)-(x)]\,dx$$

$$= \left[\frac{x^3}{3} + \frac{x^2}{2} + 2x\right]_0^1$$

$$= \frac{1}{3} + \frac{1}{2} + 2$$

$$= \frac{17}{6}$$



Area of a Region Between Intersecting Curves

Example:

Find the area of the region bounded the graph of $f(x) = 2 - x^2$, and $g(x) = x$

<u>Solution:</u> Notice in graph that there are 2 intersection points.

Let's find them: $2 - x^2 = x$

$$-x^2 - x + 2 = 0$$

$$-(x^2 + x - 2) = 0$$

$$-(x+2)(x-1) = 0$$

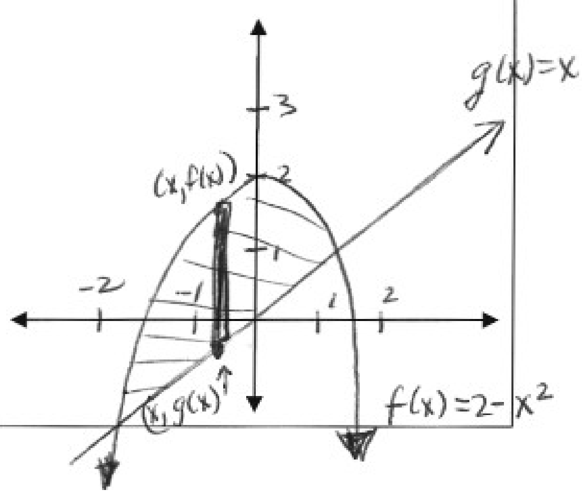
$$x = -2 \text{ or } 1$$

$$\therefore a = -2$$
$$b = 1$$

$$A = \int_{-2}^{1} [(2-x^2)-x]\,dx$$

$$= \left[-\frac{x^3}{3} - \frac{x^2}{2} + 2x\right]_{-2}^{1}$$

$$= \frac{9}{2}$$



Example: Curves that Intersect at More than Two Points

Find the area of the region bounded the graph of $f(x) = 3x^3 - x^2 - 10x$, and $g(x) = -x^2 + 2x$

Solution: Let's first find the points of intersection

$$3x^3 - x^2 - 10x = -x^2 + 2x$$

$$3x^3 - 12x = 0 \qquad\qquad x = -2, 0, 2$$

$$3x(x^2 - 4) = 0$$

$$3x(x+2)(x-2)$$

Since one function is not always $\leq$ the other,
We need two integrals

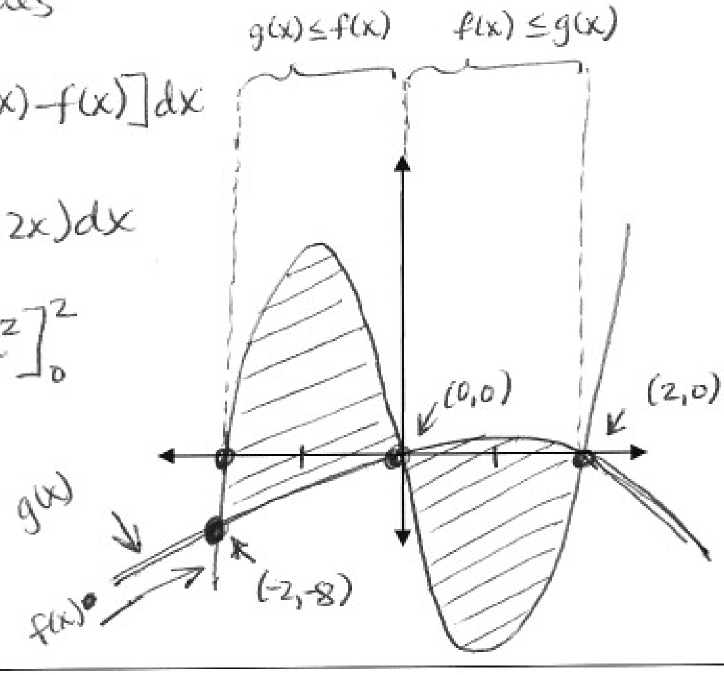
$$A = \int_{-2}^{0} [f(x) - g(x)]dx + \int_{0}^{2} [g(x) - f(x)]dx$$

$$= \int_{-2}^{0} (3x^3 - 12x)dx + \int_{0}^{2} (-3x^3 + 12x)dx$$

$$= \left[\frac{3x^4}{4} - 6x^2\right]_{-2}^{0} + \left[\frac{-3x^4}{4} + 6x^2\right]_{0}^{2}$$

$$= -(12 - 24) + (-12 + 24)$$

$$= 24$$



HW: p. 413 #1-6:all; #13-23: odd (use calculator for #19)