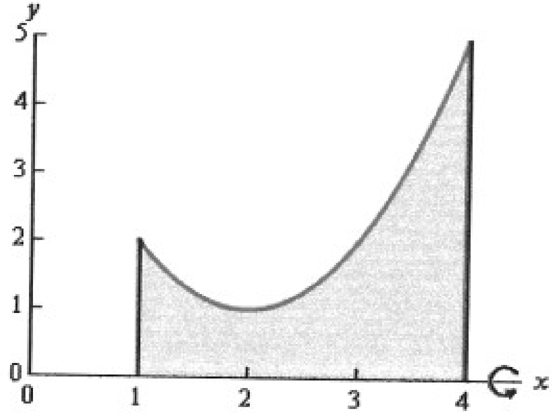
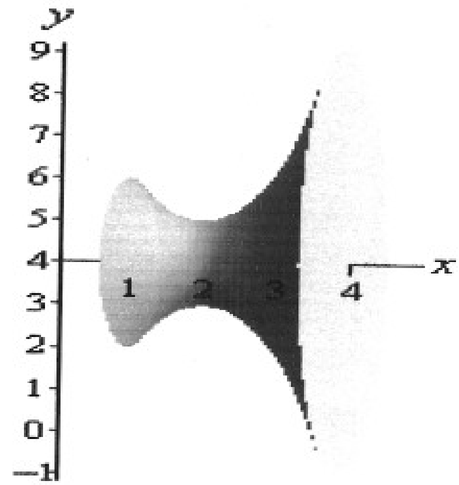
The Disc Method

Let's find the volume of 3-dimensional solids. Specifically, solids of revolution. These will have cross sections that are similar.



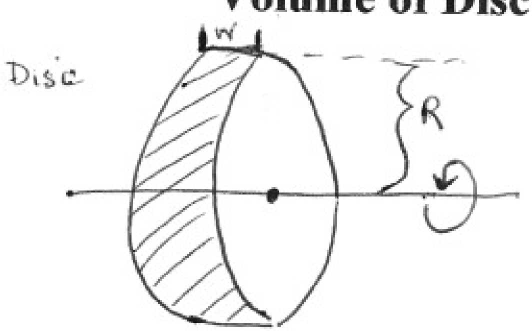


Solid of Revolution = a region revolved around a line.

Axis of Revolution = rotate around this line.

Disc = a right circular cylinder.

Volume of Disc = (area of disc)(width of disc)

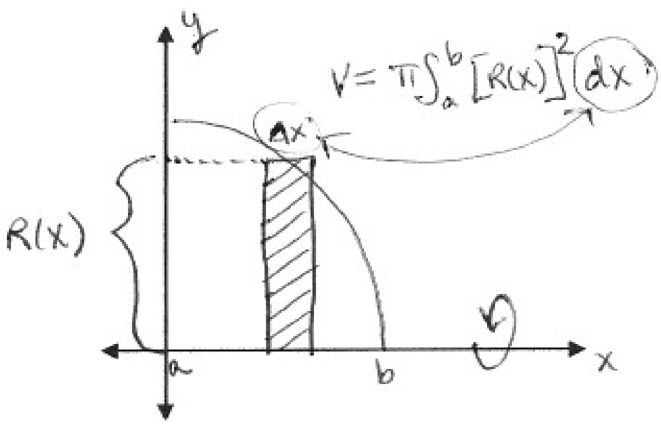


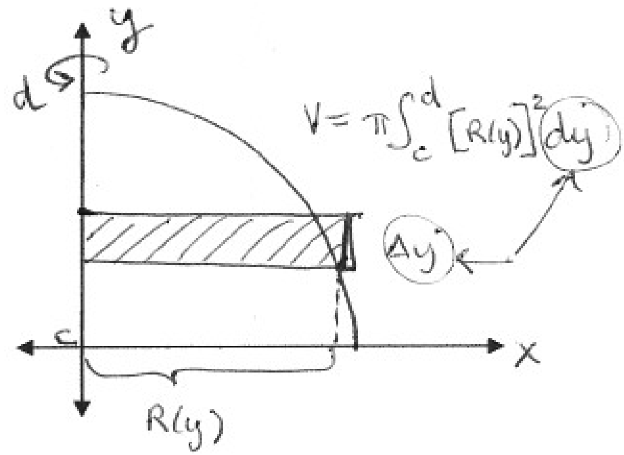
$$V = \pi R^2 w$$

The Disc Method

To find the volume of a solid of revolution with the DISC METHOD, use one of the following:

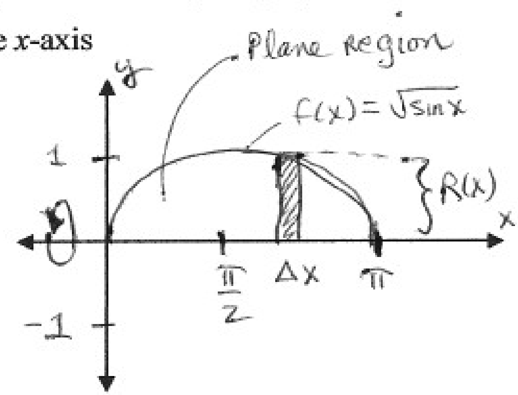
Horizontal Axis of Revolution	Vertical Axis of Revolution
$Volume = V = \pi \int_a^b [R(x)]^2 dx$	$Volume = V = \pi \int_c^d [R(y)]^2 dy$

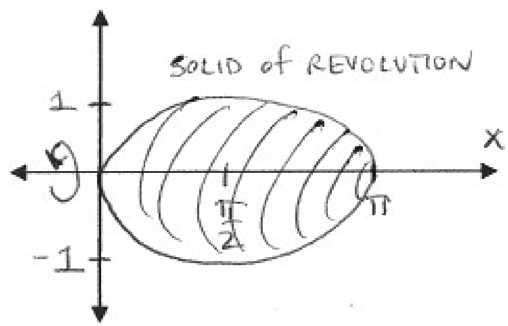




Example:

Find the volume of the solid formed by revolving the region bounded by the graph of $f(x) = \sqrt{\sin x}$ and the x-axis $(0 \le x \le \pi)$ about the x-axis





Solution:

By visual inspection of upper graph, we can see $R(x) = f(x) = \sqrt{\sin x}$

Thus,

$$V = \pi \int_a^b [R(x)]^2 dx = \pi \int_0^{\pi} (\sqrt{\sin x})^2 dx$$

$$= \pi \int_0^{\pi} \sin x \, dx$$

$$= \pi \Big[-\cos x \Big]_0^{\pi}$$

$$= \pi (1+1)$$

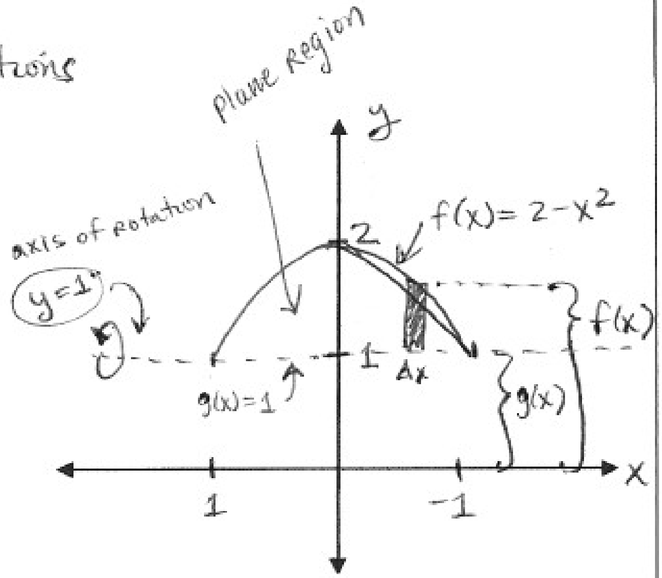
$$= 2\pi$$

Example:

Find the volume of the solid formed by revolving the region bounded by the graph of $f(x) = 2 - x^2$ and $g(x) = 1$ about the line $y = 1$

Solution:

By solving the system of equations f(x) and g(x), we find their intersection: $x = \pm 1$



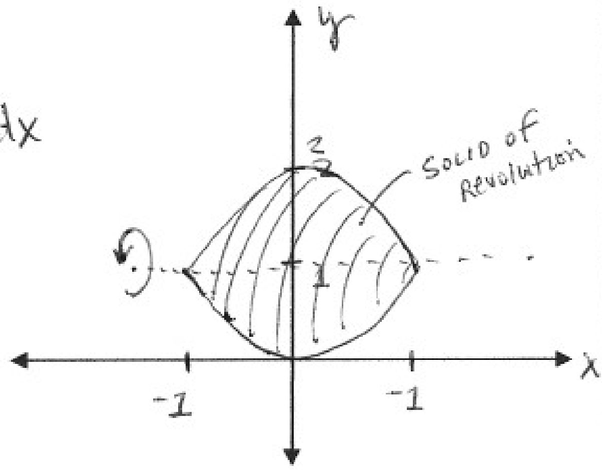
Now, find the radius of the solid by:

$$R(x) = f(x) - g(x)$$

$$= (2-x^2) - 1$$

$$= 1 - x^2$$

Then, integrate between -1 and 1 to find the volume.



$$V = \pi \int_a^b [R(x)]^2 dx = \pi \int_{-1}^{1} (1-x^2)^2 dx$$

$$= \pi \int_{-1}^{1} (1 - 2x^2 + x^4) dx$$

$$= \pi \left[x - \frac{2x^3}{3} + \frac{x^5}{5} \right]_{-1}^{1}$$

$$= \frac{16\pi}{15}$$

The Washer Method

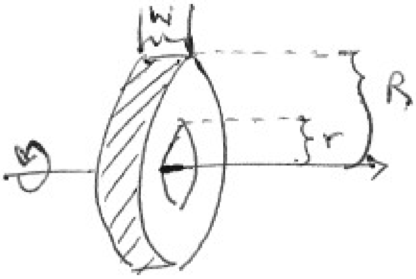
The Washer Method is a Disc with a hole cut into it.

Washer = a disc (right circular cylinder) with a hole

Outer Radius = $R(x)$

Inner Radius = $r(x)$

Volume of Washer = Disc − [(area of hole)(width of hole)]



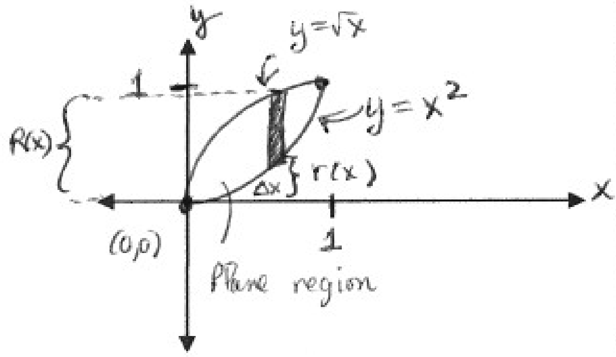
$$V = \pi R^2 w - \pi r^2 w$$

Example:

Find the volume of the solid formed by revolving the region bounded by the graphs $f(x) = \sqrt{x}$ and $f(x) = x^2$ about the x-axis

Solution:

Look at graphs to see inner and outer radii



$R(x) = \sqrt{x}$ outer radius

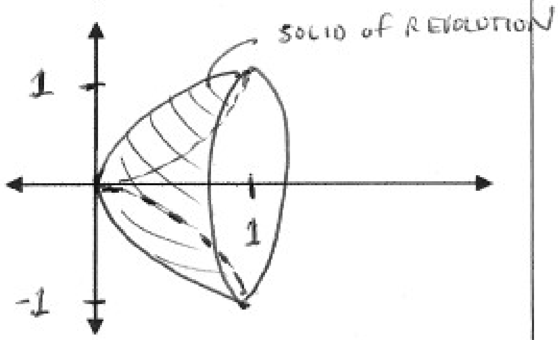
$r(x) = x^2$ Inner radius

Integrate between 0 and 1:

$$V = \pi \int_a^b \{[R(x)]^2 - [r(x)]^2\} dx$$

$$= \pi \int_0^1 (x - x^4) dx = \pi \left[\frac{x^2}{2} - \frac{x^5}{5}\right]_0^1$$

$$= \frac{3\pi}{10}$$



Example:

Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^2 + 1, y = 0, x = 0$ and $x = 1$ about the y-axis.

<u>Solution:</u>

Outer Region: $R = 1$

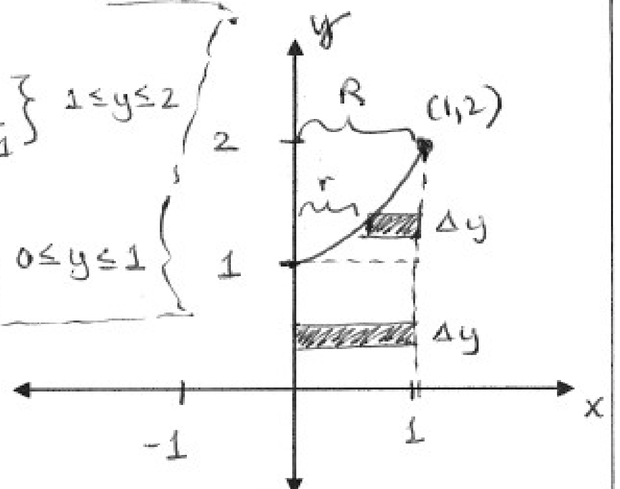
Inner Region:

$$r(y) = \begin{cases} 0 & 0 \le y \le 1 \\ \sqrt{y-1} & 1 \le y \le 2 \end{cases}$$

Notice:

$\left.\begin{array}{l} R = 1 \\ r = \sqrt{y-1} \end{array}\right\} 1 \le y \le 2$

$\left.\begin{array}{l} R = 1 \\ r = 0 \end{array}\right\} 0 \le y \le 1$



Thus, we need to break this into 2 integrals

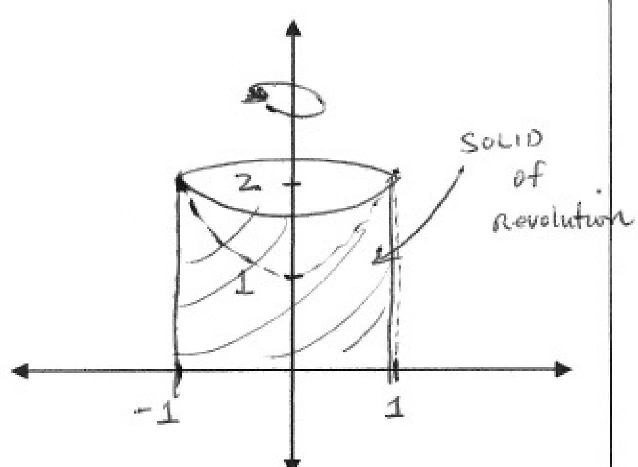
$$V = \pi \int_0^1 (1^2 - 0^2)\,dy + \pi \int_1^2 \left[1^2 - (\sqrt{y-1})^2\right] dy$$

$$= \pi \int_0^1 1\,dy + \pi \int_1^2 (2-y)\,dy$$

$$= \pi \left[y\right]_0^1 + \pi \left[2y - \frac{y^2}{2}\right]_1^2$$

$$= \pi + \pi\left(4 - 2 - 2 + \frac{1}{2}\right)$$

$$= \frac{3\pi}{2}$$



HW Part I: p. 423 #1-6:all; #11a, #12b, #31-34

HW Part II: p. 423 #7-12:all: #15-16; #19-20

(a & b, only)