Marshall Math – HW Solutions Course Calculus

#1 $\int_0^3 \sqrt{y+1}\,dy$

Let $u = y+1$
$du = 1\,dy$

LOWER	UPPER
when $y=0$	when $y=3$
$u = 0+1 = 1$	$u = 3+1 = 4$

$\int_1^4 u^{1/2}\,du = \frac{2}{3}u^{3/2}\Big]_1^4$

$= \frac{2}{3}(8) - \frac{2}{3}(1) = \frac{16}{3} - \frac{2}{3}$

$= \frac{14}{3}$

#2 $\int_0^1 r\sqrt{1-r^2}\,dr$

Let $u = 1-r^2$
$du = -2r\,dr$

$= -\frac{1}{2}\int_0^1 (-2r)\sqrt{1-r^2}\,dr$

LOWER	UPPER
when $r=0$	when $r=1$
$u = 1-0^2 = 1$	$u = 1-1^2 = 0$

$= -\frac{1}{2}\int_1^0 u^{1/2}\,du$

$= -\frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_1^0$

$= -\frac{1}{2}\left[0 - \left(\frac{2}{3}\right)(1)\right] = \frac{1}{3}$

#3 $\int_0^{\pi/4} \tan x \sec^2 x\,dx$

Let $u = \tan x$
$du = \sec^2 x\,dx$

LOWER	UPPER
when $x=0$	when $x=\pi/4$
$u = \tan 0$	$u = \tan \pi/4$
$= 0$	$= 1$

$\int_0^1 u\,du = \frac{u^2}{2}\Big]_0^1$

$= \frac{1^2}{2} - \frac{0^2}{2} = \frac{1}{2}$

#4 $\int_{-1}^1 \frac{5r}{(4+r^2)^2}\,dr$

Let $u = 4+r^2$
$du = 2r\,dr$

LOWER	UPPER
when $r=-1$	when $r=1$
$u = 4+(-1)^2$	$u = 4+(1)^2$
$= 5$	$= 5$

$\frac{5}{2}\int_5^5 \left(\frac{1}{u^2}\right)du = 0$

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#5 $\int_{-1}^{1} 3x^2\sqrt{x^3+1}\,dx$

Let $u = x^3+1$
$du = 3x^2\,dx$

LOWER: when $x=-1$, $u=(-1)^3+1=0$
UPPER: when $x=1$, $u=(1)^3+1=2$

$\int_0^2 u^{1/2}\,du = \frac{2}{3}\left[u^{3/2}\right]_0^2$

$= \left(\frac{2}{3}\right)\left[2^{3/2}-0^{3/2}\right] = \left(\frac{2}{3}\right)2\sqrt{2}$

$= \frac{4\sqrt{2}}{3}$

#6 $\int_0^1 \frac{x^3}{\sqrt{x^4+9}}\,dx$

Let $u = x^4+9$
$du = 4x^3\,dx$

LOWER: when $x=0$, $u=0^4+9=9$
UPPER: when $x=1$, $u=(1)^4+9=10$

$\frac{1}{4}\int_9^{10} u^{-1/2}\,du$

$= \frac{1}{4}(2)\,u^{1/2}\Big]_9^{10} = \frac{1}{2}\left(10^{1/2}-9^{1/2}\right)$

$= \frac{1}{2}\left(\sqrt{10}-3\right)$

#7 $\int_{-\pi}^{\pi} \frac{\cos x}{\sqrt{4+3\sin x}}\,dx$

Let $u = 4+3\sin x$
$du = 3\cos x$

LOWER: when $x=-\pi$, $u=4+3\sin(-\pi) = 4+3(0)=4$
UPPER: when $x=\pi$, $u=4+3\sin\pi = 4+3(0) = 4$

$\frac{1}{3}\int_4^4 u^{-1/2}\,du = 0$

#8 $\int_0^1 \sqrt{t^5+2t}\,(5t^4+2)\,dt$

Let $u = t^5+2t$
$du = (5t^4+2)\,dt$

LOWER: when $t=0$, $u=0^5+2(0)=0$
UPPER: when $t=1$, $u=1^5+2(1) = 1+2=3$

$\int_0^3 u^{1/2}\,du = \frac{2}{3}u^{3/2}\Big]_0^3$

$= \frac{2}{3}\left[(3)^{3/2}-(0)^{3/2}\right]$

$= \left(\frac{2}{3}\right)(3\sqrt{3}) = 2\sqrt{3}$

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#9 $\int_0^{\pi/6} \cos^{-3}(2\theta)\sin(2\theta)\,d\theta$

Let $u = \cos(2\theta)$

$du = -\sin(2\theta)\cdot 2 = -2\sin(2\theta)\,d\theta$

LOWER: When $\theta = 0$, $u = \cos(0) = 1$

UPPER: When $\theta = \pi/6$, $u = \cos(\pi/6) = \frac{1}{2}$

$$= -\frac{1}{2}\int_1^{1/2} u^{-3}\,du$$

$$= -\frac{1}{2}\left[\frac{u^{-2}}{-2}\right]_1^{1/2} = \left(+\frac{1}{4}\right)\left[\frac{1}{u^2}\right]_1^{1/2}$$

$$= \left(\frac{1}{4}\right)\left(\frac{1}{1/4} - 1\right) = \left(\frac{1}{4}\right)(3) = \frac{3}{4}$$

#10 $\int_{\pi/6}^{\pi/3} (1-\cos 3x)\sin 3x\,dx$

Let $u = 1 - \cos 3x$

$du = 3\sin 3x$

LOWER: When $x = \pi/6$, $u = 1 - \cos(3\cdot\pi/6) = 1 - \cos\frac{\pi}{2} = 1$

UPPER: When $x = \frac{\pi}{3}$, $u = 1 - \cos(3\cdot\frac{\pi}{3}) = 1 - \cos\pi = 1 - (-1) = 2$

$$= \frac{1}{3}\int_1^2 u\,du$$

$$= \frac{1}{3}\left[\frac{u^2}{2}\right]_1^2 = \frac{1}{3}\left[\frac{4}{2} - \frac{1}{2}\right]$$

$$= (1/3)(3/2) = \frac{1}{2}$$