HW (4.3) pp. 271-272 #1-9: odd; #21-27: odd

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#1 $\int_0^5 3\,dx$

#3 $\int_{-4}^{4} (4-|x|)\,dx$

OR

$\int_{-4}^{0} (x+4)\,dx + \int_0^4 (-x+4)\,dx$

#5 $\int_{-2}^{2} (4-x^2)\,dx$

#7 $\int_0^{\pi} \sin x\,dx$

#9 Notice function is interms of "y"

$\int_0^2 y^3\,dy$

#21 GIVEN: $\int_0^5 f(x)\,dx = 10$ $\int_5^7 f(x)\,dx = 3$

(a) $\int_0^7 f(x) = \int_0^5 f(x)\,dx + \int_5^7 f(x)\,dx$

$= 10 + 3 = 13$

(b) $\int_5^6 f(x)\,dx = -\int_0^5 f(x)\,dx = -10$

(c) $\int_5^5 f(x)\,dx = 0$

(d) $\int_0^5 3f(x)\,dx = 3\int_0^5 f(x)\,dx$

$= 3(10) = 30$

#23 GIVEN $\int_2^6 f(x)\,dx = 10$ $\int_2^6 g(x)\,dx = -2$

(a) $\int_2^6 [-f(x)+g(x)]\,dx = 10+(-2)$

$= 8$

(b) $\int_2^6 [g(x)-f(x)]\,dx = -2-10$

$= -12$

(c) $\int_2^6 2g(x)\,dx = 2(-2) = -4$

(d) $\int_2^6 3f(x)\,dx = 3(10) = 30$

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#25) $\int_4^{10} 6\,dx$ $\quad y = 6$ on $[4,10]$ $\quad c_i = 4 + \frac{6i}{n}$

$\Delta x = \frac{10-4}{n} = \frac{6}{n}$ $\quad \|\Delta\| \to 0$ as $n \to \infty$

$$\sum_{i=1}^{n} f(c_i)\Delta x_i = \sum_{i=1}^{n} f\left(4+\frac{6i}{n}\right)\left(\frac{6}{n}\right) = \sum_{i=1}^{n} 6\left(\frac{6}{n}\right)$$

($= 6$ as stated above)

$$= \sum_{i=1}^{n} \frac{36}{n} = 36$$

#27) $\int_{-1}^{1} x^3\,dx$ $\quad y = x^3$ on $[-1,1]$ $\quad c_i = -1 + \frac{2i}{n}$

$\Delta x = \frac{1-(-1)}{n} = \frac{2}{n}$ $\quad \|\Delta\| \to 0$ as $n \to \infty$

$$\sum_{i=1}^{n} f(c_i)\Delta x_i = \sum_{i=1}^{n} f\left(-1+\frac{2i}{n}\right)\left(\frac{2}{n}\right) = \sum_{i=1}^{n} \left(-1+\frac{2i}{n}\right)^3\left(\frac{2}{n}\right)$$

$$= \sum_{i=1}^{n} \left[-1 + \frac{6i}{n} - \frac{12i^2}{n^2} + \frac{8i^3}{n^3}\right]\left(\frac{2}{n}\right)$$

$$= -2 + \frac{12}{n^2}\sum_{i=1}^{n} i - \frac{24}{n^3}\sum_{i=1}^{n} i^2 + \frac{16}{n^4}\sum_{i=1}^{n} i^3$$

$$= -2 + \frac{12}{n^2}\left[\frac{n(n+1)}{2}\right] - \frac{24}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right] + \frac{16}{n^4}\left[\frac{n^2(n+1)^2}{4}\right]$$

$$= -2 + 6\left(1+\frac{1}{n}\right) - 4\left(2+\frac{3}{n}+\frac{1}{n^3}\right) + 4\left(1+\frac{2}{n}+\frac{1}{n^2}\right) \neq \frac{2}{n}$$