(5.1) HW pp. 318 – 319 #15-27:eoo
#41-65:odd

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#15) $\ln\frac{2}{3} = \ln 2 - \ln 3$

#19) $\ln\sqrt{2^3} = \ln 2^{3/2} = \frac{3}{2}\ln 2$

#23) $\ln z(z-1)^2$
$= \ln z + \ln(z-1)^2$
$= \ln z + 2\ln(z-1)$

#27) $\frac{1}{3}\left[2\ln(x+3) + \ln x - \ln(x^2-1)\right]$
$= \frac{1}{3}\ln\frac{(x+3)^2(x)}{(x^2-1)}$
$= \frac{1}{3}\ln\frac{x(x+3)^2}{(x^2-1)} = \ln\left[\frac{x(x+3)^2}{x^2-1}\right]^{1/3}$
$= \ln\sqrt[3]{\frac{x(x+3)^2}{x^2-1}}$

#41) $g(x) = \ln x^2 = 2\ln x$
$g'(x) = (2)\left(\frac{1}{x}\right) = \frac{2}{x}$

#43) $y = (\ln x)^4$
$y' = 4(\ln x)^3\left(\frac{1}{x}\right) = \frac{4(\ln x)^3}{x}$

#45) $y = \ln x\sqrt{x^2-1}$
$y = \ln x + \frac{1}{2}\ln(x^2-1)$
$y' = \frac{1}{x} + \left(\frac{1}{2}\right)\left(\frac{1}{x^2-1}\right)(2x)$
$= \frac{1}{x} + \frac{x}{x^2-1} = \frac{2x^2-1}{x(x^2-1)}$

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#47 $f(x) = \ln \frac{x}{x^2+1}$

$f(x) = \ln x - \ln(x^2+1)$

$f'(x) = \frac{1}{x} - \left(\frac{1}{x^2+1}\right)(2x)$

$= \frac{1-x^2}{x(x^2+1)}$

#49 $g(t) = \frac{\ln t}{t^2}$

$g'(t) = \frac{t^2(1/t) - (\ln t)(2t)}{(t^2)^2}$

$= \frac{t - 2t\ln t}{t^4} = \frac{1-2\ln t}{t^3}$

#51 $y = \ln(\ln x^2)$

$y' = \frac{1}{\ln x^2}\frac{d}{dx}(\ln x^2)$

$= \frac{1}{\ln x^2}\left(\frac{1}{x^2}\right)(2x) = \frac{2}{x\ln x^2}$

$= \frac{2}{(x)(2)\ln x} = \frac{1}{x\ln x}$

#53 $y = \ln\sqrt{\frac{x+1}{x-1}}$

$y = \frac{1}{2}\left[\ln(x+1) - \ln(x-1)\right]$

$y' = \left(\frac{1}{2}\right)\left(\frac{1}{x+1}\right)(1) - \left(\frac{1}{2}\right)\left(\frac{1}{x-1}\right)(1)$

$= \frac{1}{2}\left[\frac{1}{x+1} - \frac{1}{x-1}\right]$

$= \frac{1}{2}\left[\frac{(x-1)-(x+1)}{(x+1)(x-1)}\right]$

$= \left(\frac{1}{2}\right)\left(\frac{-2}{x^2-1}\right) = \frac{-1}{x^2-1}$

$= \frac{1}{1-x^2}$

#55 $f(x) = \ln\frac{\sqrt{4+x^2}}{x}$

$f(x) = \frac{1}{2}\ln(4+x^2) - \ln x$

$f'(x) = \left(\frac{1}{2}\right)\left(\frac{1}{4+x^2}\right)(2x) - \frac{1}{x}$

$= \frac{x}{4+x^2} - \frac{1}{x}$

$= \frac{-4}{x(x^2+4)}$

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#57) $y = \frac{-\sqrt{x^2+1}}{x} + \ln(x+\sqrt{x^2+1})$

$y' = \frac{(x)(-1)(\frac{1}{2})(x^2+1)^{-1/2}(2x) - (1)(-1)(x^2+1)^{1/2}}{x^2} + \left(\frac{1}{x+\sqrt{x^2+1}}\right)\left(1+\frac{1}{2}(x^2+1)^{-1/2}(2x)\right)$

$= \frac{-x\left(\frac{x}{\sqrt{x^2+1}}\right)+\sqrt{x^2+1}}{x^2} + \left(\frac{1}{x+\sqrt{x^2+1}}\right)\left(\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}\right)$

$= \frac{1}{x^2\sqrt{x^2+1}} + \frac{1}{\sqrt{x^2+1}}$

$= \frac{1+x^2}{x^2\sqrt{x^2+1}} = \frac{\sqrt{x^2+1}}{x^2}$

#59) $y = \ln|\sin x|$

$y' = \frac{1}{\sin x}(\cos x) = \frac{\cos x}{\sin x}$

$y' = \cot x$

#61) $y = \ln\left|\frac{\cos x}{\cos x - 1}\right|$

$y = \ln|\cos x| - \ln|\cos x - 1|$

$y' = \frac{1}{\cos x}(-\sin x) - \frac{1}{\cos x - 1}(-\sin x)$

$= -\tan x + \frac{\sin x}{\cos x - 1}$

#63) $y = \ln\left|\frac{-1+\sin x}{2+\sin x}\right|$

$y = \ln|-1+\sin x| - \ln|2+\sin x|$

$y' = \frac{1}{-1+\sin x}\cdot\cos x - \frac{1}{2+\sin x}\cdot\cos x$

$= \frac{\cos x}{\sin x - 1} - \frac{\cos x}{\sin x + 2}$

$= \frac{\cos x(\sin x+2) - \cos x(\sin x - 1)}{(\sin x - 1)(\sin x + 2)}$

$= \frac{3\cos x}{(\sin x - 1)(\sin x + 2)}$

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(65) $f(x) = \sin 2x \ln x^2$

$f(x) = \sin 2x \cdot (2) \ln x$

$= 2 \sin 2x \ln x$

$f'(x) = 2\left[\sin 2x \left(\frac{1}{x}\right) + \cos 2x (2) \ln x\right]$

$= \frac{2}{x} \sin 2x + 4 \cos 2x \ln x$

$= \frac{2}{x}(\sin 2x + 2x \cos 2x \ln x)$

$= \frac{2}{x}(\sin 2x + x \cos 2x \ln x^2)$