(6.2) HW: Part II p.423 #7-12: all (a & b, only)
#15-16; #19-20

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#7 $y = x^2 \Rightarrow x = \sqrt{y}$

(Revolve around y-axis)

$V = \pi \int_0^4 (\sqrt{y})^2 \, dy$

$= \pi \int_0^4 y \, dy = \pi \left[\frac{y^2}{2}\right]_0^4$

$= 8\pi$

#8 $y = \sqrt{16 - x^2} \Rightarrow x = \sqrt{16 - y^2}$

$V = \pi \int_0^4 \left(\sqrt{16 - y^2}\right)^2 dy$

$= \pi \int_0^4 (16 - y^2) \, dy$

$= \pi \left[16y - \frac{y^3}{3}\right]_0^4$

$= \frac{128\pi}{3}$

#9 $y = x^{2/3} \longrightarrow x = y^{3/2}$

$V = \pi \int_0^1 (y^{3/2})^2 dy$

$= \pi \int_0^1 y^3 \, dy = \pi \left[\frac{y^4}{4}\right]_0^1$

$= \frac{\pi}{4}$

#10 $x = -y^2 + 4y$

$V = \pi \int_1^4 (-y^2 + 4y)^2 dy$

$= \pi \int_1^4 (y^4 - 8y^3 + 16y^2) \, dy$

$= \pi \left[\frac{y^5}{5} - 2y^4 + \frac{16y^3}{3}\right]_1^4$

$= \frac{459\pi}{15} = \frac{153\pi}{5}$

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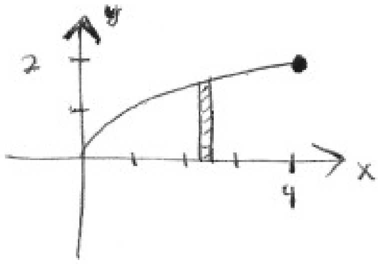
#11 $y=\sqrt{x}$ $\quad y=0,\ x=4$

(a) $R(x)=\sqrt{x}$, $r(x)=0$

$$V=\pi\int_0^4 (\sqrt{x})^2\,dx=\left[\frac{\pi}{2}x^2\right]_0^4$$

$$=8\pi$$

DISC (about x-axis)



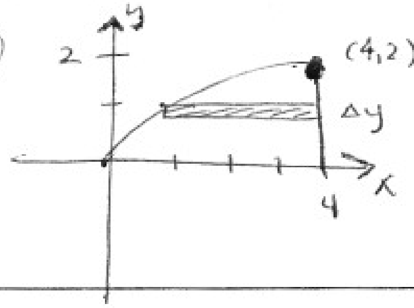
(b) $R(y)=4$, $r(y)=y^2$

$$V=\pi\int_0^2\left[4^2-(y^2)^2\right]dy$$

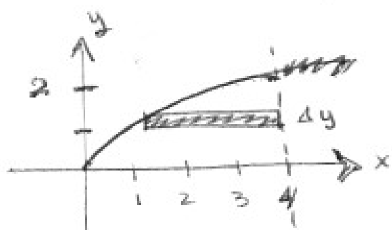
$$=\pi\int_0^2 (16-y^4)\,dy$$

$$=\pi\left[16y-\frac{1}{5}y^5\right]_0^2=\frac{128\pi}{5}$$

Washer



(c) The line $x=4$



when $x=4 \Rightarrow y=2$

$$V=\pi\int_0^2\left[(4)-y^2\right]^2dy$$

$$=\pi\int_0^2 (16-8y^2+y^4)\,dy$$

$$=\pi\left[16y-\frac{8}{3}y^3+\frac{1}{5}y^5\right]_0^2$$

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#12 $y = 2x^2 \quad y = 0 \quad x = 2$

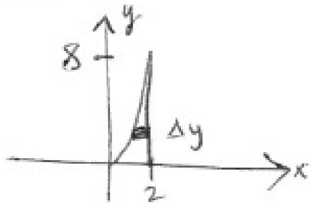
(a) $R(y) = 2$

$r(y) = \sqrt{y/2}$ (about y-axis)

Washer

$$V = \int_0^8 \left[2^2 - \left(\sqrt{y/2}\right)^2\right] dy$$

$$= \pi\left[4y - \frac{y^2}{4}\right]_0^8 = 16\pi$$



(b) (about x-axis)

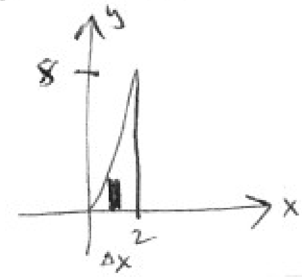
$R(x) = 2x^2$

$r(x) = 0$

Disc

$$V = \pi\int_0^2 \left[2x^2\right]^2 dx$$

$$= \pi\left[\frac{4x^5}{5}\right]_0^2 = \frac{128\pi}{5}$$



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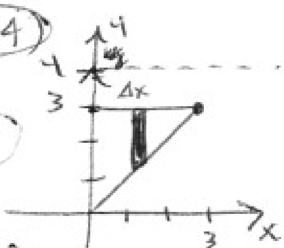
#15 $y=x \quad y=3 \quad x=0$

(about line $y=4$)

$f(x)=4 \quad g(x)=x$



$R(x)=f(x)-g(x)=4-x$

$r(x)=1$

washer

$$V=\pi\int_0^3 [R(x)]^2-[r(x)]^2\,dx$$

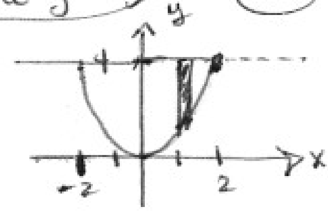
$$=\pi\int_0^3 [(4-x)^2-(1)^2]\,dx$$

$$=\pi\int_0^3 (x^2-8x+15)\,dx$$

$$=\pi\left[\frac{x^3}{3}-4x^2+15x\right]_0^3=18\pi$$

#16 $y=x^2 \quad y=4$ ~~washer~~ Disc

(about line $y=4$)



$R(x)=f(x)-g(x)$
$=4-x^2$

$r(x)=0 \Leftarrow$ disc

$$V=\pi\int_{-2}^{2}(4-x^2)^2\,dx$$

$$V=2\pi\int_0^2(4-x^2)^2\,dx$$

$$=2\pi\int_0^2(x^4-8x^2+16)\,dx$$

$$=2\pi\left[\frac{x^5}{5}-\frac{8x^3}{3}+16x\right]_0^2$$

$$=\frac{512\pi}{15}$$

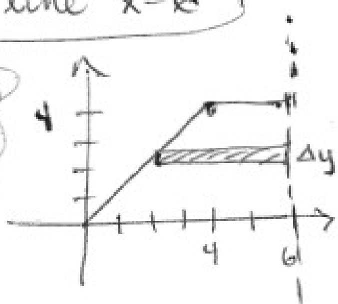
#19 $y=x \quad y=0 \quad y=4 \quad x=6$

(about line $x=6$)

$R(y)=f(y)-g(y)$
$=6-y$

$r(y)=0$

disc



$$V=\pi\int_0^4 [R(y)]^2\,dy$$

$$=\pi\int_0^4 (6-y)^2\,dy$$

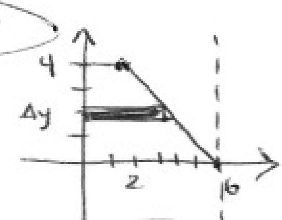
$$=\pi\int_0^4 (y^2-12y+36)\,dy=\frac{208}{3}\pi$$

#20 $y=6-x,\ y=0 \quad y=4 \quad x=0$

(about line $x=6$)

$R(y)=6$

$r(y)=6-(6-y)$
$=y$



washer

$$V=\pi\int_0^4 [(6)^2-(y)^2]\,dy$$

$$=\pi\left[36y-\frac{y^3}{3}\right]_0^4=\frac{368\pi}{3}$$