

1. A car comes to a stop five seconds after the driver slams on the brakes. While the brakes are on, the following velocities are recorded.

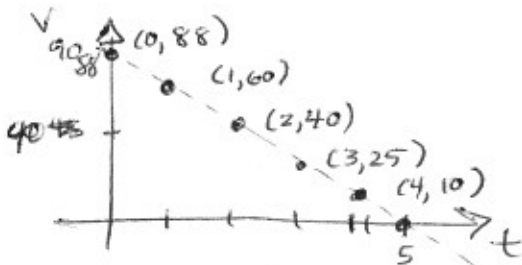
Time since brakes applied (sec)	0	1	2	3	4	5
Velocity (ft/sec)	88	60	40	25	10	0

- a. Give lower and upper estimates of the distance the car traveled after the brakes were applied.

$$88 + 60 + 40 + 25 + 10 = 223 \text{ feet}$$

$$60 + 40 + 25 + 10 + 0 = 135 \text{ feet}$$

- b. On a sketch of velocity against time, show the lower and upper estimates.



2. Roger decides to run a marathon. Roger's friend Jeff rides behind him on a bicycle and clocks his pace every 15 minutes. Roger starts out strong, but after an hour and a half he is so exhausted that he has to stop. The data Jeff collected is summarized below.

Time spent running (min)	0	15	30	45	60	75	90
Speed (mph)	12	11	10	10	8	7	0

- a. Assuming that Roger's speed is always decreasing, give upper and lower estimates for the distance Roger ran during the first half hour.

$$\Delta t = \frac{1}{4} \text{ hour}$$

$$\frac{1}{4} (12 + 11) = 5.75 \text{ miles}$$

$$\frac{1}{4} (11 + 10) = 5.25 \text{ miles}$$

- b. Give upper and lower estimates for the distance Roger ran in total.

$$\frac{1}{4} (12 + 11 + 10 + 10 + 8 + 7) = 14.5 \text{ miles}$$

$$\frac{1}{4} (11 + 10 + 10 + 8 + 7 + 0) = 11.5 \text{ miles}$$

3. Coal gas is produced at a gasworks. Pollutants in the gas are removed by scrubbers, which become less and less efficient as time goes on. Measurements made at the start of each month showing the rate at which pollutants are escaping in the gas are as follows.

Time (months)	0	1	2	3	4	5	6
Rate pollutants are escaping (tons/month)	5	7	8	10	13	16	20

- a. Make an overestimate and an underestimate of the total quantity of pollutants that escaped during the first month.

$$\Delta t = 1$$

$$1(5) = 5 \text{ tons}$$

$$1(7) = 7 \text{ tons}$$

- b. Make an overestimate and an underestimate of the total quantity of pollutants that escaped during the first six months.

$$1(5+7+8+10+13+16) = 59 \text{ tons}$$

$$1(7+8+10+13+16+20) = 74 \text{ tons}$$

- 4-7. Use upper and lower sums to approximate the area bounded by each curve and the x-axis over the indicated interval using 4 equal subdivisions.

4. $\int_0^2 (4-x^2) dx$

$$\frac{b-a}{n} = \frac{2-0}{4} = \frac{1}{2}$$

0 $4-0^2 = 4$

$\frac{1}{2}$ $4-(\frac{1}{2})^2 = 3.75$

1 $4-(1)^2 = 3$

$\frac{3}{2}$ $4-(\frac{3}{2})^2 = 1.75$

2 $4-(2)^2 = 0$

$$\frac{1}{2} (4 + 3.75 + 3 + 1.75) = 6.25$$

$$\frac{1}{2} (3.75 + 3 + 1.75 + 0) = 4.25$$

5. $\int_0^1 (x^3) dx$

$$\frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4}$$

0 $0^3 = 0$

$\frac{1}{4}$ $(\frac{1}{4})^3 = \frac{1}{64}$

$\frac{1}{2}$ $(\frac{1}{2})^3 = \frac{1}{8}$

$\frac{3}{4}$ $(\frac{3}{4})^3 = \frac{27}{64}$

1 $(1)^3 = 1$

$$\frac{1}{4} (0 + \frac{1}{64} + \frac{1}{8} + \frac{27}{64}) = \frac{9}{64} \approx 0.14$$

$$\frac{1}{4} (\frac{1}{64} + \frac{1}{8} + \frac{27}{64} + 1) = \frac{25}{64} \approx 0.39$$

Calculus : Integrals

Riemann Sums

Name KEY

Date _____ Period _____

6. $\int_0^{\frac{\pi}{2}} (\cos x) dx = \frac{\frac{\pi}{2} - 0}{4} = \frac{\pi}{8}$

7. $\int_1^2 (\frac{1}{x}) dx = \frac{2-1}{4} = \frac{1}{4}$

0 $\cos 0 = 1$

1 $\frac{1}{1} = 1$

$\frac{\pi}{8}$ $\cos \frac{\pi}{8} = 0.924$

$\frac{5}{4}$ $\frac{4}{5}$

$\frac{\pi}{4}$ $\cos \frac{\pi}{4} = 0.707$

$\frac{6}{4}$ $\frac{4}{6} = \frac{2}{3}$

$\frac{3\pi}{8}$ $\cos \frac{3\pi}{8} = 0.383$

$\frac{7}{4}$ $\frac{4}{7}$

$\frac{\pi}{2}$ $\cos \frac{\pi}{2} = 0$

2 $\frac{1}{2}$

$\frac{\pi}{8} (1 + 0.924 + 0.707 + 0.383) = 1.184$

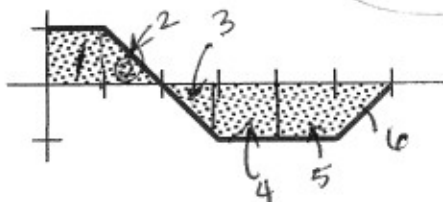
$\frac{1}{4} (1 + \frac{4}{5} + \frac{2}{3} + \frac{4}{7}) = 0.760$

$\frac{\pi}{8} (0.924 + 0.707 + 0.383 + 0) = 0.791$

$\frac{1}{4} (\frac{4}{5} + \frac{2}{3} + \frac{4}{7} + \frac{1}{2}) = 0.635$

8. The graph of f is shown to the right. Find the area bound by the curve and the x -axis over the interval $[0, 6]$.

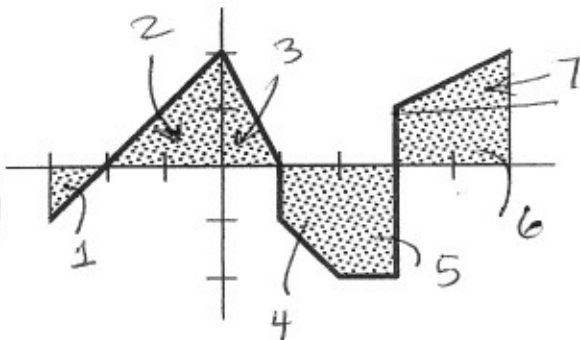
units
 $1 + \frac{1}{2} - \frac{1}{2} - 1 - 1 - \frac{1}{2} = -1\frac{1}{2}$



9. The graph of g is shown to the right.

a. Find the area bounded by the x -axis and $g(x)$ over the interval $[-3, 5]$.

$-\frac{1}{2} + 2 + \frac{1}{2} - \frac{3}{2} - 2 + 2 + 1 = 2$



b. Find $\int_{-3}^5 g(x) dx$.

$= 2$