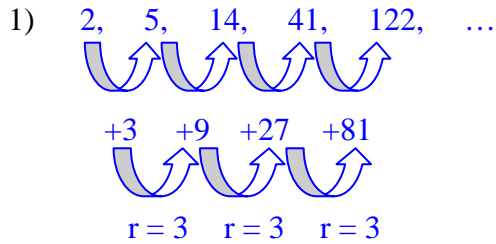


Solutions: Review for Fall Semester Final

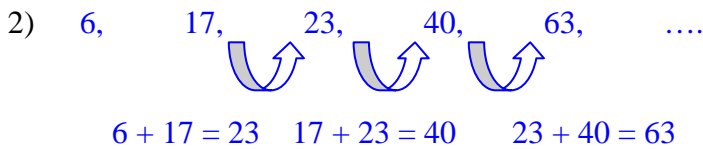
(Stuff in blue is helpful hints/thinking, but not stuff you need to write.)



This sequence represents mixed recursion.

$$\begin{cases} a_1 = 2 \\ a_n = 3a_{n-1} - 1 \end{cases}$$

2, 5, 14, 41, 122, 365, 1094, 3281, ...



This sequence is a Fibonacci sequence.

$$\begin{cases} a_1 = 6 \\ a_2 = 17 \\ a_n = a_{n-1} + a_{n-2} \end{cases}$$

6, 17, 23, 40, 63, 103, 166, 269, ...

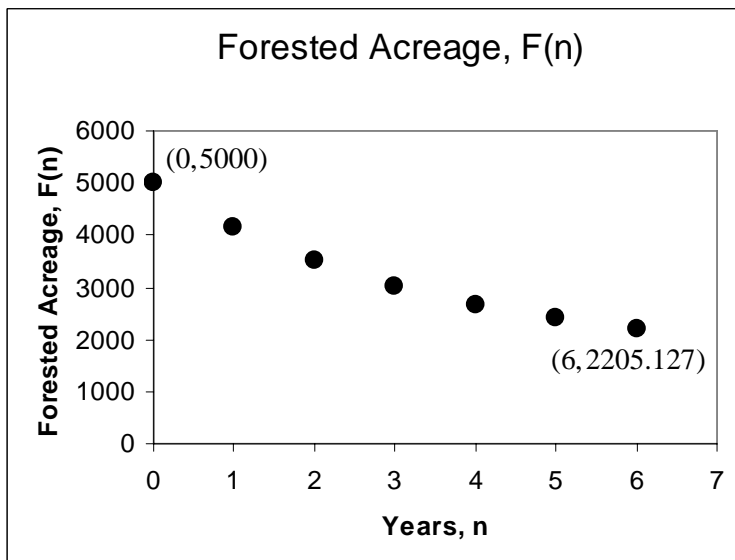
3) let F_n = the amount of forested acreage after n years

$$\begin{cases} F_0 = 5000 \\ F_n = 0.75F_{n-1} + 400 \end{cases}$$

Remember that losing 25% means keeping 75%.

(losing: 100% - lost% increasing: 100% + gain%)

4)



5)

n	$F(n)$
0	5000
10	1791.465950
20	1610.782121
30	1600.607179
40	1600.034192

In the long run, the amount of forested acreage approaches 1600 acres.

6) let A_n = the savings account balance (\$) after n months

$$\begin{cases} A_0 = 1500 \\ A_n = (1 + \frac{.0075}{12})A_{n-1} + 50 \end{cases}$$

Even when using Excel for the solutions, show variables and formulas.

$$A_{18} = 2421.76$$

The savings account balance is \$2,421.76 after 18 months.

7) Be sure to show a verbal model.

$$\begin{aligned} \text{accrued interest} &= \text{balance} - [\text{principal} + (\text{monthly dep})(\# \text{ months})] \\ &= 2421.76 - [1500 + (50)(18)] \\ &= 21.76 \end{aligned}$$

Of the balance after 18 months, \$21.76 represents accrued interest.

8) let A_n = the mortgage balance (\$) after n months

$$\begin{cases} A_0 = 235000 \\ A_n = (1 + \frac{.065}{12})A_{n-1} - pmt \end{cases}$$

$$\text{if } pmt = 1485.36, \text{ then } A_{360} = 0.25$$

$$\text{if } pmt = 1485.37, \text{ then } A_{360} = -10.90$$

To repay the mortgage in 30 years requires a monthly payment of \$1,485.37.

9)

$$\begin{aligned} \text{total paid} &= (\text{monthly pmt})(\# \text{ of months}) - \text{overpmt} \\ &= 1485.37(360) - 10.90 \\ &= 534722.3 \end{aligned}$$

You pay a total of \$534,722.30 for the mortgage.

10) let B_n = the mortgage balance (\$) after n months

$$\begin{cases} B_0 = 235000 \\ B_n = (1 + \frac{.065}{12})B_{n-1} - 1560.37 \end{cases}$$

Need a new variable and recursive formula.

$$B_{313} = 223.63 \text{ and } B_{314} = -1335.53$$

$$\begin{aligned} \text{new total paid} &= (\text{monthly pmt})(\# \text{ of months}) - \text{overpmt} \\ &= 1560.37(314) - 1335.53 \\ &= 488620.65 \end{aligned}$$

$$\begin{aligned} \text{savings} &= \text{original total paid} - \text{new total paid} \\ &= 534722.30 - 488620.65 \\ &= 46101.65 \end{aligned}$$

Paying an extra \$75 per month, you save \$46,101.65 in interest.

$$11) \begin{cases} a_0 = 256 \\ a_n = -\frac{3}{4}a_{n-1} \end{cases}$$

$$12) a_n = 256\left(-\frac{3}{4}\right)^n$$

13) This question is not clear. The 7th partial sum starting at $n = 0$ means go to $n = 6$. Skip this question.

$$14) \sum_{n=0}^{\infty} 256\left(-\frac{3}{4}\right)^n = \frac{256}{1 - \left(-\frac{3}{4}\right)} = \frac{256}{\frac{7}{4}} = 256\left(\frac{4}{7}\right) = \frac{1024}{7}$$

$$15) \sum_{j=1}^{21} \frac{3j+7}{5} = \frac{21(2+14)}{2} = 168$$

$$a_j = \frac{3j+7}{5}$$

$$a_1 = 2$$

$$a_{21} = 14$$

Since this is an arithmetic sequence, you need to show work/formulas.

$$16) \frac{7}{1} + \frac{12}{1} + \frac{17}{2} + \frac{22}{6} + \frac{27}{24} + \frac{32}{120} + \frac{37}{720} + \dots = \sum_{i=0}^{\infty} \frac{5i+7}{i!} \approx 32.619382$$

n	$\sum_{i=0}^n \frac{5i+7}{i!}$
5	32.558333
10	32.619380
20	32.619382
50	32.619382

The trick here is to first see that the denominator is factorials, BUT starting at $i = 0$. The numerator is arithmetic; now we use the formula starting at $i = 0$.

$$17) \sum_{n=0}^{\infty} 4\left(\frac{7}{2}\right)^n \text{ does not exist, since } |r| = \left|\frac{7}{2}\right| > 1$$

$$18) 5 - \frac{6}{4} + \frac{7}{9} - \frac{8}{16} + \frac{9}{25} - \frac{10}{36} + \dots - \frac{16}{144} = \sum_{i=1}^{12} \frac{i+4}{i^2} \approx 9.363$$

This is neither arithmetic nor geometric, so simply use the calculator to find the sum.

19) convenience sampling

20) stratified sampling

21) The variable is *the response* about number of different species documented in the past year by a NMSB member at the annual meeting. The population is *the set of responses* about number of different species documented in the past year by a NMSB member from all members at the annual meeting. The sample is *the set of responses* about number of different species documented in the past year by a NMSB member from the 20 sampled members.

22) nominal

23) ratio

24) ordinal

25) interval

26) ratio

27) sampling

28) simulation

29) census

30) experiment