Half-Life Worksheet **Name:** Key

Half-life is defined to be the time required for something to fall to half its initial value (in particular, the time for half the atoms in a radioactive substance to disintegrate).

1. Let Q represent a mass of radioactive Radium, in grams, whose half-life is 1620 years. The quantity of radium present after t years is given by $Q(t) = 25\left(\frac{1}{2}\right)^{t/1620}$.

   a. Determine the initial quantity of Radium.

   $Q(0) = 25$

   The initial quantity of Radium is 25 grams.

   b. Determine the quantity present after 1000 years. After 1620 years. After 3240 years.

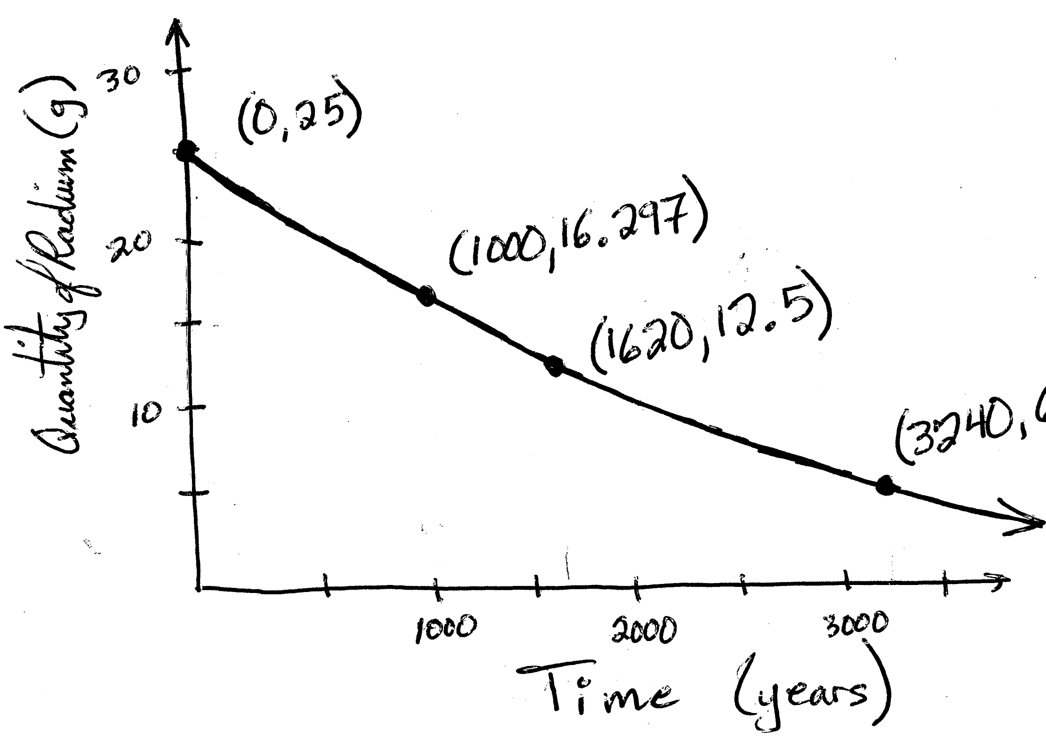
   $Q(1000) = 25\left(\frac{1}{2}\right)^{1000/1620} \approx 16.297$

   $Q(1620) = 25\left(\frac{1}{2}\right)^{1620/1620} = 25\left(\frac{1}{2}\right)^{1} = 12.5$

   $Q(3240) = 25\left(\frac{1}{2}\right)^{3240/1620} = 25\left(\frac{1}{2}\right)^{2} = 6.25$

   The amount of Radium present after 1000 years is 16.297 grams, after 1620 years is 12.5 grams, and after 3240 years is 6.25 grams.

   c. Use a graphing calculator to graph the function on the interval $0 \le t \le 5000$. Sketch below.



d. When will the quantity of Radium be approximately 20 grams? How did you determine this?

solve $Q(t)=20$

$t \approx 521.524$

The quantity of Radium will be 20 grams after 521.5 years.

e. When will the quantity of Radium be 0 grams? Explain.

The quantity of Radium will never be 0 grams since an exponential function does not intersect its horizontal asymptote, which is $y=0$ in this case.

2. Let Q represent a mass of Carbon-14, in grams, whose half-life is 5730 years. The quantity present after t years is given by $Q(t)=10\left(\frac{1}{2}\right)^{t/5730}$.

a. Determine the initial quantity of Carbon-14.

$Q(0)=10\left(\frac{1}{2}\right)^{0/5730}=10$

The initial quantity of Carbon-14 is 10 grams.

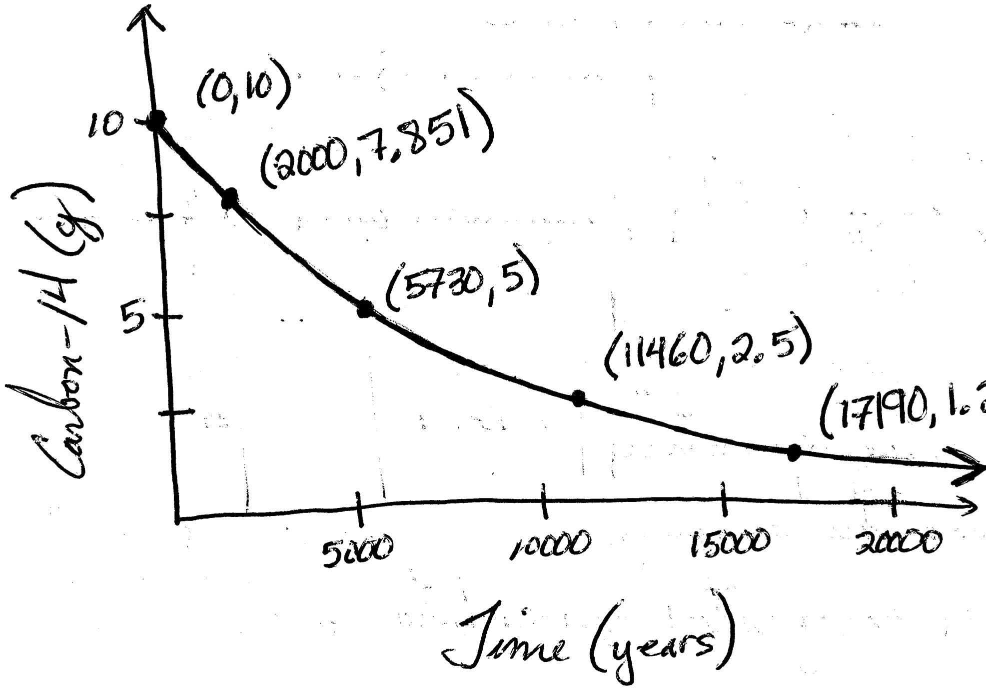
b. Determine the quantity present after 2000 years. After 5730 years.

$Q(2000)\approx 7.851$

$Q(5730)=10\left(\frac{1}{2}\right)^{5730/5730}=10\left(\frac{1}{2}\right)^{1}=5$

The quantity of Carbon-14 will be 7.851 grams after 2000 years and 5 grams after 5730 years.

c. Sketch the graph of the function over the interval $0 \le t \le 10{,}000$.



3. In both of the problems, the equation for the quantity present after *t* years followed the form $Q(t) = a \cdot \left(\frac{1}{2}\right)^{t/c}$.

a. Why is ½ the "natural" choice for the base of the half-life exponential function?

Because the half-life describes how long it takes for half of a substance to decay.

b. Look at the multiplier out front, *a*. In the problems above, what did *a* represent in the problem? Why?

The coefficient a represents the initial quantity of the substance because $\left(\frac{1}{2}\right)^0$ always equals 1, meaning $Q(0) = a\left(\frac{1}{2}\right)^0 = a$.

c. In the exponent in both problems, *t* was divided by a number *c* (or multiplied by a number 1/*c*). In each problem, what did *c* represent? What kind of function transformation is this?

The number c represents the half-life of the substance. This results in a transformation given by a horizontal stretch of the graph.

d. What you should notice is that if you know the initial quantity and the half-life, you can write a function describing the quantity left after a certain time. Use this fact to write a function describing the quantity of Ununquadium-289 left after *t* seconds if the initial quantity was 7 grams. Ununquadium-289 has a half-life of 30 seconds.

$$Q(t) = 7\left(\frac{1}{2}\right)^{t/30}$$

e. Look back at your equation from the Candyons experiment. Given that the half-life of an M&M was one trial, use the form above to write an equation for the quantity left after a certain number of trials. How close is it to the equation you made in class? Write both below, graph both on a calculator, and discuss.