(1) $$\lim_{x\to 2}\frac{x^2-4}{x^3-8} = \lim_{x\to 2}\frac{(x-2)(x+2)}{(x-2)(x^2+2x+4)}$$

$$= \lim_{x\to 2}\frac{x+2}{x^2+2x+4} = \frac{4}{4+4+4} = \frac{1}{3}$$

(2) $$\lim_{x\to 4}\frac{\sqrt{2x}-\sqrt{x+4}}{4-x} = \lim_{x\to 4}\left[\frac{\sqrt{2x}-\sqrt{x+4}}{4-x}\cdot\frac{\sqrt{2x}+\sqrt{x+4}}{\sqrt{2x}+\sqrt{x+4}}\right]$$

$$= \lim_{x\to 4}\frac{2x-(x+4)}{(4-x)(\sqrt{2x}+\sqrt{x+4})}$$

$$= \lim_{x\to 4}\frac{x-4}{(4-x)(\sqrt{2x}+\sqrt{x+4})}$$

$$= \lim_{x\to 4}\frac{-1}{\sqrt{2x}+\sqrt{x+4}} = \frac{-1}{\sqrt{8}+\sqrt{8}}$$

$$= \frac{-1}{4\sqrt{2}} = \frac{-\sqrt{2}}{8}$$

(3) $$\lim_{x\to 0}\frac{\frac{3}{4-x^2}-\frac{3}{4}}{\frac{1}{2-x}-\frac{1}{2}} = \lim_{x\to 0}\left[\frac{\frac{3}{4-x^2}-\frac{3}{4}}{\frac{1}{2-x}-\frac{1}{2}}\cdot\frac{4(4-x^2)}{4(2-x)(2+x)}\right]$$

$$= \lim_{x\to 0}\frac{12-3(4-x^2)}{4(2+x)-2(4-x^2)}$$

$$= \lim_{x\to 0}\frac{3x^2}{8+4x-8+2x^2}$$

$$= \lim_{x\to 0}\frac{3x^2}{2x(x+2)}$$

$$= \lim_{x\to 0}\frac{3x}{2(x+2)} = 0$$

④ $\lim_{x\to -2} \frac{3x^2+5x+1}{x^2-4}$ DNE

⑤ $\lim_{x\to 0} \frac{x\sin x}{x+\sin x} = \lim_{x\to 0}\left[\frac{x\sin x}{x+\sin x}\cdot\frac{\frac{1}{x}}{\frac{1}{x}}\right]$

$= \lim_{x\to 0} \frac{\sin x}{1+\frac{\sin x}{x}}$

$= \frac{\lim_{x\to 0}\sin x}{\lim_{x\to 0} 1 + \lim_{x\to 0}\frac{\sin x}{x}} = \frac{0}{1+1} = 0$

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⑤ $\lim_{x\to 0} \frac{\sin x}{2x} = \frac{1}{2}\lim_{x\to 0}\frac{\sin x}{x} = \frac{1}{2}\cdot 1 = \frac{1}{2}$

⑪ $\lim_{t\to 0} \frac{\tan^2 3t}{2t} = \frac{1}{2}\lim_{t\to 0} \frac{\frac{\sin 3t}{\cos 3t}\cdot\frac{\sin 3t}{\cos 3t}}{t}$

$= \frac{1}{2}\lim_{t\to 0}\left(\frac{\sin 3t}{t}\cdot\frac{3}{3}\right)\cdot\lim_{t\to 0}\frac{\sin 3t}{\cos^2 3t}$

★ as $t\to 0$, $3t\to 0$

$= \frac{3}{2}\lim_{3t\to 0}\frac{\sin 3t}{3t}\cdot\lim_{t\to 0}\frac{\sin 3t}{\cos^2 3t}$

$= \frac{3}{2}\cdot 1\cdot 0 = 0$

⑫ $\lim_{t\to 0}\frac{\tan 2t}{\sin 2t - 1} = \frac{0}{-1} = 0$

(13) $\lim\limits_{t\to 0} \dfrac{\sin(3t)+4t}{t\sec t} = \lim\limits_{t\to 0}\left[\dfrac{\sin 3t}{t\sec t} + \dfrac{4t}{t\sec t}\right]$

$= \lim\limits_{t\to 0}\left(\dfrac{\sin 3t}{t}\cdot\dfrac{3}{3}\right)\cdot \lim\limits_{t\to 0}\dfrac{1}{\sec t} + \lim \dfrac{4t}{t\sec t}$

★ as $t\to 0$,
$3t\to 0$

$= 3\lim\limits_{3t\to 0}\dfrac{\sin 3t}{3t}\cdot \lim\limits_{t\to 0}\dfrac{1}{\sec t} + \lim\limits_{t\to 0}\dfrac{4}{\sec t}$

$= 3\cdot 1\cdot 1 + 4 = 7$

(71) $\lim\limits_{x\to 0}\dfrac{1+\sin x}{\cos x} = \dfrac{1+0}{1} = 1$

(86) $\lim\limits_{x\to 0}\dfrac{\sin 2x}{2x^2+x} = \lim\limits_{x\to 0}\left[\dfrac{\sin 2x}{2x^2+x}\cdot\dfrac{\frac{1}{x}}{\frac{1}{x}}\right]$

$= \lim\limits_{x\to 0}\dfrac{\frac{\sin 2x}{x}}{2x+1}$

$= \dfrac{\lim\limits_{x\to 0}\left(\frac{\sin 2x}{x}\cdot\frac{2}{2}\right)}{\lim\limits_{x\to 0}(2x+1)}$

$= \dfrac{2\lim\limits_{2x\to 0}\frac{\sin 2x}{2x}}{1} = \dfrac{2\cdot 1}{1} = 2$

55 $\lim_{x\to 1} \frac{1-\sqrt{x}}{1-x} = \lim_{x\to 1} \left[\frac{1-\sqrt{x}}{1-x} \cdot \frac{1+\sqrt{x}}{1+\sqrt{x}} \right]$

$= \lim_{x\to 1} \frac{1-x}{(1-x)(1+\sqrt{x})} = \lim_{x\to 1} \frac{1}{1+\sqrt{x}} = \frac{1}{2}$

56 $\lim_{x\to 1} \frac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3} = \lim_{x\to 1} \left[\frac{(2x-3)(\sqrt{x}-1)}{(2x+3)(x-1)} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} \right]$

$= \lim_{x\to 1} \frac{(2x-3)(x-1)}{(2x+3)(x-1)(\sqrt{x}+1)}$

$= \lim_{x\to 1} \frac{2x-3}{(2x+3)(\sqrt{x}+1)} = \frac{-1}{5(2)} = -\frac{1}{10}$

49 $\lim_{x\to -5} \frac{x^2+3x-10}{x+5} = \lim_{x\to -5} \frac{(x+5)(x-2)}{x+5}$

$= \lim_{x\to -5} (x-2) = -7$

26 $\lim_{x\to \frac{1}{16}} \frac{x^{1/2}-\frac{1}{4}}{x^{1/4}-\frac{1}{2}} = \lim_{x\to \frac{1}{16}} \frac{(x^{1/4}-\frac{1}{2})(x^{1/4}+\frac{1}{2})}{x^{1/4}-\frac{1}{2}}$

$= \lim_{x\to \frac{1}{16}} \left(x^{1/4}+\frac{1}{2}\right) = \frac{1}{2}+\frac{1}{2} = 1$

(26) $$\lim_{x\to 0} \frac{\sin^2 x}{1-\cos x} = \lim_{x\to 0}\left[\frac{\sin^2 x}{1-\cos x}\cdot\frac{1+\cos x}{1+\cos x}\right]$$

$$= \lim_{x\to 0} \frac{\sin^2 x\,(1+\cos x)}{1-\cos^2 x}$$

$$= \lim_{x\to 0} \frac{\sin^2 x\,(1+\cos x)}{\sin^2 x}$$

$$= \lim_{x\to 0} (1+\cos x) = 2$$

(28) $$\lim_{x\to 0} \frac{1-\cos 3x}{\sin 3x} = \lim_{x\to 0}\left(\frac{1-\cos 3x}{\sin 3x}\cdot\frac{3x}{3x}\right)$$

$$= \lim_{x\to 0} \frac{1-\cos 3x}{3x} \cdot \lim_{x\to 0} \frac{3x}{\sin 3x}$$

$$= \lim_{3x\to 0} \frac{1-\cos 3x}{3x} \cdot \lim_{3x\to 0} \frac{3x}{\sin 3x}$$

as $x\to 0$,
$3x \to 0$

$$= 0\cdot 1 = 0$$

(29) $$\lim_{x\to 0} \frac{2x-\cot x}{x+3\cot x} = \lim_{x\to 0}\left[\frac{2x-\frac{\cos x}{\sin x}}{x+3\frac{\cos x}{\sin x}}\cdot\frac{\sin x}{\sin x}\right]$$

$$= \lim_{x\to 0} \frac{2x\sin x - \cos x}{x\sin x + 3\cos x}$$

$$= \frac{-1}{3}$$

(30) $\lim_{x\to 0} \dfrac{\cos x - \sec x}{1 - \sec x} = \lim_{x\to 0} \left[\dfrac{\cos x - \frac{1}{\cos x}}{1 - \frac{1}{\cos x}} \cdot \dfrac{\cos x}{\cos x} \right]$

$$= \lim_{x\to 0} \frac{\cos^2 x - 1}{\cos x - 1}$$

$$= \lim_{x\to 0} \frac{(\cos x - 1)(\cos x + 1)}{\cos x - 1}$$

$$= \lim_{x\to 0} (\cos x + 1) = 2$$