More Limits Practice WS

Pg 1 C #49–53, 55, 56, 58
D #48, 49

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(49) $\lim_{x \to a} \frac{x^2 - a^2}{x - a} = \lim_{x \to a} \frac{(x-a)(x+a)}{x-a} = \lim_{x \to a} (x+a) = 2a$

(+2) format (+4) factoring/alg (+1) parentheses
(+3) answer

(50) $\lim_{x \to a} \frac{x^2 - a^2}{x + a} = \frac{0}{2a} = 0$

(−2) factored/cancelled unnecessarily

(51) $\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$

$= \lim_{h \to 0} \frac{2xh + h^2}{h} = \lim_{h \to 0} (2x + h) = 2x$

(−3) bad, evil alg (−2) skipped steps
(−3) skipped All-But-1-Pt Thrm step

(52) $\lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \lim_{h \to 0} \left(\frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \right)$

$= \lim_{h \to 0} \frac{x + h - x}{h(\sqrt{x+h} + \sqrt{x})}$

$= \lim_{h \to 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$

(53) $$\lim_{\Delta x \to 0} \frac{\frac{1}{x+\Delta x} - \frac{1}{x}}{\Delta x} = \lim_{\Delta x \to 0} \left(\frac{\frac{1}{x+\Delta x} - \frac{1}{x}}{\Delta x} \cdot \frac{x(x+\Delta x)}{x(x+\Delta x)} \right)$$

$$= \lim_{\Delta x \to 0} \frac{x - (x+\Delta x)}{x(\Delta x)(x+\Delta x)}$$

$$= \lim_{\Delta x \to 0} \frac{-\Delta x}{x(\Delta x)(x+\Delta x)}$$

$$= \lim_{\Delta x \to 0} \frac{-1}{x(x+\Delta x)} = -\frac{1}{x^2}$$

(55) $$\lim_{x \to 1} \frac{1-\sqrt{x}}{1-x} = \lim_{x \to 1} \left(\frac{1-\sqrt{x}}{1-x} \cdot \frac{1+\sqrt{x}}{1+\sqrt{x}} \right)$$

$$= \lim_{x \to 1} \frac{1-x}{(1-x)(1+\sqrt{x})} = \lim_{x \to 1} \frac{1}{1+\sqrt{x}} = \frac{1}{2}$$

(56) $$\lim_{x \to 1} \frac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3} = \lim_{x \to 1} \left[\frac{(2x-3)(\sqrt{x}-1)}{(2x+3)(x-1)} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} \right]$$

$$= \lim_{x \to 1} \frac{(2x-3)(x-1)}{(2x+3)(x-1)(\sqrt{x}+1)}$$

$$= \lim_{x \to 1} \frac{2x-3}{(2x+3)(\sqrt{x}+1)} = \frac{-1}{5(2)} = \frac{-1}{10}$$

(58) $$\lim_{x \to 1} \frac{\sqrt{x+1} - \sqrt{2x}}{x^2 - x} = \lim_{x \to 1} \left[\frac{\sqrt{x+1} - \sqrt{2x}}{x(x-1)} \cdot \frac{\sqrt{x+1} + \sqrt{2x}}{\sqrt{x+1} + \sqrt{2x}} \right]$$

$$= \lim_{x \to 1} \frac{x+1-2x}{x(x-1)(\sqrt{x+1} + \sqrt{2x})}$$

$$= \lim_{x \to 1} \frac{1-x}{x(x-1)(\sqrt{x+1} + \sqrt{2x})}$$

$$= \lim_{x \to 1} \frac{-1}{x(\sqrt{x+1} + \sqrt{2x})} = -\frac{1}{2\sqrt{2}} = \frac{-\sqrt{2}}{4}$$

(48) $\lim_{x \to 2} \frac{x-2}{x^2+5} = \frac{0}{9} = 0$

(49) $\lim_{x \to -5} \frac{x^2+3x-10}{x+5} = \lim_{x \to -5} \frac{(x-2)(x+5)}{x+5}$

$= \lim_{x \to -5} (x-2) = -7$

LH Pg 104 #11, 12, 18–26, 74–76

More Limit Practice WS 2 # 20, 27, 29, 34, 35, 36

Bonus

(11) a) $\lim_{x\to 0} f(x) = 2$ b) $\lim_{x\to 1} f(x) = 3$

(12) a) $\lim_{x\to 0} f(x) = 1$ b) $\lim_{x\to \pi/2} f(x)$ DNE

do not put = sign

(18) $\lim_{x\to 2} \frac{3x+5}{5x-3} = \frac{11}{7}$

(19) $\lim_{t\to 3} \frac{t^2+1}{t} = \frac{10}{3}$

these two are not indeterminate (you can just "plug in")

(20) $\lim_{t\to 3} \frac{t^2-9}{t-3} = \lim_{t\to 3} \frac{(t-3)(t+3)}{t-3}$

$= \lim_{t\to 3} (t+3) = 6$

must show orig prob first (before factoring or multiplying by a special form of 1)

must show step after cancelling BEFORE giving answer (also, the parentheses are not optional here with the addition)

(21) $\lim_{t\to -2} \frac{t+2}{t^2-4} = \lim_{t\to -2} \frac{t+2}{(t+2)(t-2)}$

$= \lim_{t\to -2} \frac{1}{t-2} = -\frac{1}{4}$

(22) $\lim_{x\to 0} \frac{\sqrt{4+x}-2}{x} = \lim_{x\to 0} \left[\frac{\sqrt{4+x}-2}{x} \cdot \frac{\sqrt{4+x}+2}{\sqrt{4+x}+2}\right]$ ← use brackets

$= \lim_{x\to 0} \frac{4+x-4}{x(\sqrt{4+x}+2)} = \lim_{x\to 0} \frac{x}{x(\sqrt{4+x}+2)}$

$= \lim_{x\to 0} \frac{1}{\sqrt{4+x}+2} = \frac{1}{4}$

show orig prob first

take your time with the steps

show step after cancelling

(23) $\lim_{x\to 0} \dfrac{\frac{1}{x+1}-1}{x} = \lim_{x\to 0}\left[\dfrac{\frac{1}{x+1}-1}{x}\cdot\dfrac{x+1}{x+1}\right]$

$= \lim_{x\to 0} \dfrac{1-(x+1)}{x(x+1)}$ ← use parentheses so you remember to distribute

$= \lim_{x\to 0} \dfrac{-x}{x(x+1)}$ ← simplify the numerator first, then cancel with denominator (take your time with the steps)

$= \lim_{x\to 0} -\dfrac{1}{x+1}$

$= -1$

(24) $\lim_{s\to 0} \dfrac{\frac{1}{\sqrt{1+s}}-1}{s} = \lim_{s\to 0}\left[\dfrac{\frac{1}{\sqrt{1+s}}-1}{s}\cdot\dfrac{\sqrt{1+s}}{\sqrt{1+s}}\right]$ ← gets rid of complex fraction

$= \lim_{s\to 0}\left[\dfrac{1-\sqrt{1+s}}{s(\sqrt{1+s})}\cdot\dfrac{1+\sqrt{1+s}}{1+\sqrt{1+s}}\right]$ ← rationalize numerator

$= \lim_{s\to 0} \dfrac{1-(1+s)}{s\sqrt{1+s}\,(1+\sqrt{1+s})}$

$= \lim_{s\to 0} \dfrac{-s}{s\sqrt{1+s}\,(1+\sqrt{1+s})}$

$= \lim_{s\to 0} \dfrac{-1}{\sqrt{1+s}\,(1+\sqrt{1+s})} = -\dfrac{1}{2}$

(25) $\lim_{x\to -5} \dfrac{x^3+125}{x+5} = \lim_{x\to -5} \dfrac{(x+5)(x^2-5x+25)}{(x+5)}$

$= \lim_{x\to -5} (x^2-5x+25)$ ← parentheses not optional here

$= 75$

(26) $\displaystyle\lim_{x\to -2} \frac{x^2-4}{x^3+8} = \lim_{x\to -2} \frac{(x-2)(x+2)}{(x+2)(x^2-2x+4)}$

$\displaystyle = \lim_{x\to -2} \frac{x-2}{x^2-2x+4} = \frac{-4}{12} = \frac{-1}{3}$

(74) $\displaystyle\lim_{x\to 0} x^3 = 0$ True $\left(\text{since } \lim_{x\to c} [f(x)]^3 = [\lim_{x\to c} f(x)]^3 \text{ and } \lim_{x\to c} x = c\right)$

(75) If $f(x) = g(x)$ for all real numbers other than $x=0$, and $\displaystyle\lim_{x\to 0} f(x) = L$, then $\displaystyle\lim_{x\to 0} g(x) = L$.

True (All-But-One-Pt Thrm)

(76) If $\displaystyle\lim_{x\to c} f(x) = L$, then $f(c) = L$.

False. $\displaystyle\lim_{x\to 1} \frac{x^2-1}{x-1} = 2$ but $f(1)$ undefined

(20) $\displaystyle\lim_{x\to 0} \frac{\sin^2 x}{3x^2} = \frac{1}{3} \lim_{x\to 0} \frac{\sin x}{x} \cdot \lim_{x\to 0} \frac{\sin x}{x} = \frac{1}{3}\cdot 1 \cdot 1 = \frac{1}{3}$

"pull out" constant multiplier (scalar multiple rule)

break apart in a useful way

(27) $\displaystyle\lim_{\theta\to 0} \frac{\theta^2}{1-\cos\theta} = \lim_{\theta\to 0} \left[\frac{\theta^2}{1-\cos\theta} \cdot \frac{1+\cos\theta}{1+\cos\theta}\right]$

(a+b)(a-b) gives a^2-b^2

$\displaystyle = \lim_{\theta\to 0} \frac{\theta^2(1+\cos\theta)}{1-\cos^2\theta}$

Pythagorean Identity

$\displaystyle = \lim_{\theta\to 0} \frac{\theta^2(1+\cos\theta)}{\sin^2\theta}$

show the separation of the limits

$\displaystyle = \lim_{\theta\to 0} \frac{\theta}{\sin\theta} \cdot \lim_{\theta\to 0} \frac{\theta}{\sin\theta} \cdot \lim_{\theta\to 0} (1+\cos\theta) = 2$

(29) $\lim_{\theta \to 0} \frac{\theta}{\cos\theta} = \frac{0}{1} = 0$ "plug in" first

(34) $\lim_{x \to 0} \frac{x^2 - 3\sin x}{x} = \lim_{x \to 0} \frac{x^2}{x} - \lim_{x \to 0} \frac{3\sin x}{x}$ pull apart fraction correctly

$= \lim_{x \to 0} x - 3 \lim_{x \to 0} \frac{\sin x}{x}$

$= 0 - 3 \cdot 1 = -3$

show step after cancelling

scalar multiple rule

(35) $\lim_{x \to 0} \frac{2x + \sin x}{x} = \lim_{x \to 0} \frac{2x}{x} + \lim_{x \to 0} \frac{\sin x}{x}$

$= \left(\lim_{x \to 0} 2\right) + 1 = 2 + 1 = 3$

<u>must</u> show step after cancelling (making use of All-But-One-Pt Thrm)

can go ahead and evaluate limit for part of problem if you have shown the necessary steps first

(36) Bonus

LH Pg 81 #43-63 odds

GRADE = 100 - n

(43) $g(x) = -\frac{2x^2+x}{x}$

a) $\lim_{x\to 0} g(x) = 1$ (+2)

b) $\lim_{x\to -1} g(x) = 3$ (+2)

$g(x) = \frac{-2x^2+x}{x} = \frac{x(-2x+1)}{x}$

Let $f(x) = -2x+1$.

(+2) Then $g(x) = f(x)$ for all $x \neq 0$.

(45) $g(x) = \frac{x^3-x}{x-1}$

a) $\lim_{x\to 1} g(x) = 2$ (+2)

b) $\lim_{x\to -1} g(x) = 0$ (+2)

(+2) $g(x) = \frac{x^3-x}{x-1} = \frac{x(x^2-1)}{x-1} = \frac{x(x-1)(x+1)}{x-1}$

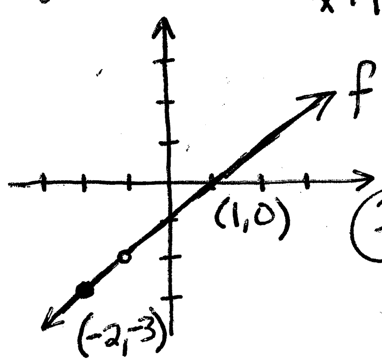
Let $f(x) = x(x+1) = x^2+x$.

Then $g(x) = f(x)$ for all $x \neq 1$.

(47) $\lim_{x\to -1} \frac{x^2-1}{x+1} = \lim_{x\to -1} \frac{(x+1)(x-1)}{x+1} = \lim_{x\to -1}(x-1) = -2$ (+3)

Let $f(x) = \frac{x^2-1}{x+1}$ and $g(x) = x-1$. Then $f(x) = g(x)$ for all $x \neq -1$.

(+2)



(+2)

$a^3+b^3 = (a+b)(a^2-ab+b^2)$ (+2)

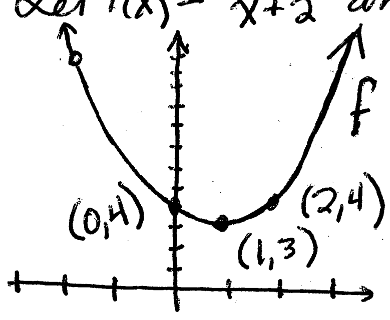
$a^3-b^3 = (a-b)(a^2+ab+b^2)$

(+2)

(49) $\lim_{x\to -2} \frac{x^3+8}{x+2} = \lim_{x\to -2} \frac{(x+2)(x^2-2x+4)}{x+2} = \lim_{x\to -2}(x^2-2x+4) = 12$ (+3)

Let $f(x) = \frac{x^3+8}{x+2}$ and $g(x) = x^2-2x+4$. Then $f(x) = g(x)$ for all $x \neq -2$.

(+2)



(+2)

LH Pg81 #43–63 odds

(51) $\lim_{x\to -4} \frac{(x+4)\ln(x+6)}{x^2-16} = \lim_{x\to -4} \frac{(x+4)\ln(x+6)}{(x+4)(x-4)} = \lim_{x\to -4} \frac{\ln(x+6)}{x-4} = \frac{-\ln 2}{8}$ +3

Let $f(x) = \frac{(x+4)\ln(x+6)}{x^2-16}$ and $g(x) = \frac{\ln(x+6)}{x-4}$.

Then $f(x) = g(x)$ for all $x \neq -4$. +2

Bonus
Graph +2

(53) $\lim_{x\to 5} \frac{x-5}{x^2-25} = \lim_{x\to 5} \frac{x-5}{(x-5)(x+5)} = \lim_{x\to 5} \frac{1}{x+5} = \frac{1}{10}$ +3 answer, +1 notation

(55) $\lim_{x\to 1} \frac{x^2+x-2}{x^2-1} = \lim_{x\to 1} \frac{(x-1)(x+2)}{(x-1)(x+1)} = \lim_{x\to 1} \frac{x+2}{x+1} = \frac{3}{2}$ +3 +1

(57) $\lim_{x\to 0} \frac{\sqrt{3+x}-\sqrt{3}}{x} = \lim_{x\to 0}\left[\frac{\sqrt{3+x}-\sqrt{3}}{x} \cdot \frac{\sqrt{3+x}+\sqrt{3}}{\sqrt{3+x}+\sqrt{3}}\right]$ +3 +1

$= \lim_{x\to 0} \frac{3+x-3}{x(\sqrt{3+x}+\sqrt{3})} = \lim_{x\to 0} \frac{1}{\sqrt{3+x}+\sqrt{3}} = \frac{1}{2\sqrt{3}}$

(59) $\lim_{x\to 0} \frac{\frac{1}{2+x}-\frac{1}{2}}{x} = \lim_{x\to 0}\left[\frac{\frac{1}{2+x}-\frac{1}{2}}{x} \cdot \frac{2(2+x)}{2(2+x)}\right] = \lim_{x\to 0} \frac{2-(2+x)}{2x(2+x)}$

$= \lim_{x\to 0} \frac{-1}{2(2+x)} = -\frac{1}{4}$

+3 +1

(61) $\lim_{\Delta x\to 0} \frac{2(x+\Delta x)-2x}{\Delta x} = \lim_{\Delta x\to 0} \frac{2x+2\Delta x-2x}{\Delta x} = \lim_{\Delta x\to 0} \frac{2\Delta x}{\Delta x} = \lim_{\Delta x\to 0} 2 = 2$

+3 +1

(63) $\lim_{\Delta x\to 0} \frac{(x+\Delta x)^2-2(x+\Delta x)+1-(x^2-2x+1)}{\Delta x}$

$= \lim_{\Delta x\to 0} \frac{\cancel{x^2}+2x(\Delta x)+(\Delta x)^2-\cancel{2x}-2\Delta x+\cancel{1}-\cancel{x^2}+\cancel{2x}-\cancel{1}}{\Delta x}$

$= \lim_{\Delta x\to 0} \frac{2x(\Delta x)+(\Delta x)^2-2\Delta x}{\Delta x}$ +3

$= \lim_{\Delta x\to 0} (2x+\Delta x-2) = 2x-2$ +1

Pg 81 # 53–58, 60, 66, 67

−2 each = grade

(53)

√ format/notation √ factored correctly √ showed step for A-B-1-Pt thrm (sign)

(54) $\lim\limits_{x \to 2} \dfrac{2-x}{x^2-4} = \lim\limits_{x \to 2} \dfrac{2-x}{(x-2)(x+2)} = \lim\limits_{x \to 2} \dfrac{-1}{x+2} = -\dfrac{1}{4}$ √√√ answer

(55)

√ format/notation √ multiplied by conjugate (attempt)

(56) $\lim\limits_{x \to 0} \dfrac{\sqrt{2+x}-\sqrt{2}}{x} = \lim\limits_{x \to 0} \left[\dfrac{\sqrt{2+x}-\sqrt{2}}{x} \cdot \dfrac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}}\right]$

$= \lim\limits_{x \to 0} \dfrac{2+x-2}{x(\sqrt{2+x}+\sqrt{2})} = \lim\limits_{x \to 0} \dfrac{\cancel{x}}{\cancel{x}(\sqrt{2+x}+\sqrt{2})}$

$= \lim\limits_{x \to 0} \dfrac{1}{\sqrt{2+x}+\sqrt{2}}$ √ showed step for A-B-1-Pt Thm

$= \dfrac{1}{\sqrt{2}+\sqrt{2}} = \dfrac{1}{2\sqrt{2}} = \dfrac{\sqrt{2}}{4}$ √√√ answers

√ format/notation

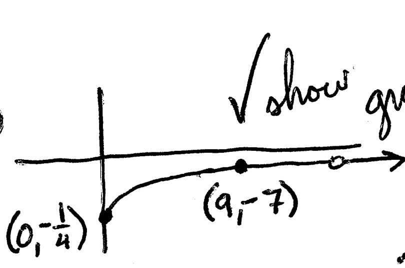
(57)

(58) $\lim\limits_{x \to 0} \dfrac{\frac{1}{x+4}-\frac{1}{4}}{x} = \lim\limits_{x \to 0} \left[\dfrac{\frac{1}{x+4}-\frac{1}{4}}{x} \cdot \dfrac{4(x+4)}{4(x+4)}\right]$ √ mult by LCD (attempt)

$= \lim\limits_{x \to 0} \dfrac{4-(x+4)}{4x(x+4)} = \lim\limits_{x \to 0} \dfrac{-\cancel{x}}{4\cancel{x}(x+4)}$ √ showed step for A-B-1-Pt Thm

$= \lim\limits_{x \to 0} \dfrac{-1}{4(x+4)} = -\dfrac{1}{16}$ √√√ answers

(66) √ show graph w/pts



let $f(x) = \dfrac{4-\sqrt{x}}{x-16}$ √ showed table

x	15.9	15.999	15.99999	16	16.00001	16.001	16.1
$f(x)$	−.125196	−.125002	−.12500002	undefined	−.12499998	−.124998	−.124805

The graph and the table both suggest $\lim\limits_{x \to 16} \dfrac{4-\sqrt{x}}{x-16} = -\dfrac{1}{8}$.

OVER →

(66) cont.

$$\lim_{x \to 16} \frac{4-\sqrt{x}}{x-16} = \lim_{x \to 16} \left[\frac{4-\sqrt{x}}{x-16} \cdot \frac{4+\sqrt{x}}{4+\sqrt{x}} \right]$$

✓ format/notation

✓ mult by conjugate (attempt)

$$= \lim_{x \to 16} \frac{16-x}{(x-16)(4+\sqrt{x})} = \lim_{x \to 16} \frac{-1}{4+\sqrt{x}} = -\frac{1}{8}$$

✓ showed step for A-B-1-Pt Thrm

✓✓✓ answer

(67)

Pg81-82 #73-90 all CALC

(75) $\lim_{x\to 0} \frac{1-e^{-x}}{e^x-1} = \lim_{x\to 0}\left(\frac{1-\frac{1}{e^x}}{e^x-1}\cdot\frac{e^x}{e^x}\right) = \lim_{x\to 0}\frac{e^x-1}{(e^x-1)e^x} = \lim_{x\to 0}\frac{1}{e^x} = 1$

(76) $\lim_{x\to 0}\frac{4(e^{2x}-1)}{e^x-1} = \lim_{x\to 0}\frac{4(e^x-1)(e^x+1)}{e^x-1} = \lim_{x\to 0} 4(e^x+1) = 8$

(78) $\lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{\sin x-\cos x} = \lim_{x\to\frac{\pi}{4}}\left(\frac{1-\frac{\sin x}{\cos x}}{\sin x-\cos x}\cdot\frac{\cos x}{\cos x}\right) = \lim_{x\to\frac{\pi}{4}}\frac{\cos x-\sin x}{(\sin x-\cos x)\cos x}$

$= \lim_{x\to\frac{\pi}{4}}\frac{-1}{\cos x}$

$= -\frac{1}{\frac{\sqrt{2}}{2}} = -\sqrt{2}$

(80) $\lim_{\phi\to 0}\phi\sec\phi = -\pi$

(81) $\lim_{x\to 6}\frac{\ln(x-5)}{\ln(x-5)^2} = \lim_{x\to 6}\frac{\ln(x-5)}{2\ln(x-5)} = \lim_{x\to 6}\frac{1}{2} = \frac{1}{2}$

(82) $\lim_{x\to 1}\frac{\ln x^2}{\ln x^3} = \lim_{x\to 1}\frac{2\ln x}{3\ln x} = \lim_{x\to 1}\frac{2}{3} = \frac{2}{3}$

(88) $\lim_{x\to\ln 2}\frac{e^{3x}-8}{e^{2x}-4} = \lim_{x\to\ln 2}\frac{(e^x-2)(e^{2x}+2e^x+4)}{(e^x-2)(e^x+2)} = \lim_{x\to\ln 2}\frac{e^{2x}+2e^x+4}{e^x+2}$

Table (−4) Graph (−2) (−1) not 2-exact pts and/or ln2 not marked $= \frac{4+4+4}{2+2} = 3$

(−4) alg/limit (−1) notation unless big (−2)

(−5 each) complete others

Solutions – Pg 84 #45 – 48, 51 – 54

+3 format/notation   +4 factoring   +2 parentheses   +3 answer

45) $\lim\limits_{x\to -1}\dfrac{x^2-1}{x+1}=\lim\limits_{x\to -1}\dfrac{(x+1)(x-1)}{x+1}=\lim\limits_{x\to -1}(x-1)=-2$

46) $\lim\limits_{x\to -1}\dfrac{2x^2-x-3}{x+1}=\lim\limits_{x\to -1}\dfrac{(x+1)(2x-3)}{x+1}=\lim\limits_{x\to -1}(2x-3)=-5$

47) $\lim\limits_{x\to 2}\dfrac{x^3-8}{x-2}=\lim\limits_{x\to 2}\dfrac{(x-2)(x^2+2x+4)}{x-2}=\lim\limits_{x\to 2}(x^2+2x+4)=12$

48) $\lim\limits_{x\to -1}\dfrac{x^3+1}{x+1}=\lim\limits_{x\to -1}\dfrac{(x+1)(x^2-x+1)}{x+1}=\lim\limits_{x\to -1}(x^2-x+1)=3$

51) $\lim\limits_{x\to 5}\dfrac{x-5}{x^2-25}=\lim\limits_{x\to 5}\dfrac{x-5}{(x-5)(x+5)}=\lim\limits_{x\to 5}\dfrac{1}{x+5}=\dfrac{1}{10}$

52) $\lim\limits_{x\to 3}\dfrac{3-x}{x^2-9}=\lim\limits_{x\to 3}\dfrac{3-x}{(x-3)(x+3)}=\lim\limits_{x\to 3}\dfrac{-1}{x+3}=-\dfrac{1}{6}$

53) $\lim\limits_{x\to -3}\dfrac{x^2+x-6}{x^2-9}=\lim\limits_{x\to -3}\dfrac{(x+3)(x-2)}{(x+3)(x-3)}=\lim\limits_{x\to -3}\dfrac{x-2}{x-3}=\dfrac{5}{6}$

54) $\lim\limits_{x\to 3}\dfrac{x^2-x-6}{x^2-5x+6}=\lim\limits_{x\to 3}\dfrac{(x-3)(x+2)}{(x-3)(x-2)}=\lim\limits_{x\to 3}\dfrac{x+2}{x-2}=5$

53. $\lim\limits_{x\to 5}\dfrac{x-5}{x^2-25}=\lim\limits_{x\to 5}\dfrac{x-5}{(x-5)(x+5)}=\lim\limits_{x\to 5}\dfrac{1}{x+5}=\dfrac{1}{10}$

54. $\lim\limits_{x\to 2}\dfrac{2-x}{x^2-4}=\lim\limits_{x\to 2}\dfrac{2-x}{(x-2)(x+2)}=\lim\limits_{x\to 2}\dfrac{-1}{x+2}=-\dfrac{1}{4}$

55. $\lim\limits_{x\to 1}\dfrac{x^2+x-2}{x^2-1}=\lim\limits_{x\to 1}\dfrac{(x+2)(x-1)}{(x-1)(x+1)}=\lim\limits_{x\to 1}\dfrac{x+2}{x+1}=\dfrac{3}{2}$

59.

$$\begin{aligned}\lim_{x\to 0}\frac{[1/(2+x)]-(1/2)}{x}&=\lim_{x\to 0}\left[\frac{\frac{1}{2+x}-\frac{1}{2}}{x}\cdot\frac{2(2+x)}{2(2+x)}\right]\\&=\lim_{x\to 0}\frac{2-(2+x)}{2x(2+x)}\\&=\lim_{x\to 0}\frac{2-2-x}{2x(2+x)}\\&=\lim_{x\to 0}\frac{-x}{2x(2+x)}\\&=\lim_{x\to 0}\frac{-1}{2(2+x)}=-\frac{1}{4}\end{aligned}$$

61. $\lim\limits_{\Delta x\to 0}\dfrac{2(x+\Delta x)-2x}{\Delta x}=\lim\limits_{\Delta x\to 0}\dfrac{2x+2(\Delta x)-2x}{\Delta x}=\lim\limits_{\Delta x\to 0}\dfrac{2(\Delta x)}{\Delta x}=\lim\limits_{\Delta x\to 0}2=2$