More Limit Practice WS

Pg 1

A.

In Problems 1–13, evaluate each limit.

1. $\lim_{x\to 0} \dfrac{\cos x}{x+1}$
2. $\lim_{\theta\to \pi/2} \theta\cos\theta$
3. $\lim_{t\to 0} \dfrac{\cos^2 t}{1+\sin t}$
4. $\lim_{x\to 0} \dfrac{3x\tan x}{\sin x}$
5. $\lim_{x\to 0} \dfrac{\sin x}{2x}$
6. $\lim_{\theta\to 0} \dfrac{\sin 3\theta}{2\theta}$
7. $\lim_{\theta\to 0} \dfrac{\sin 3\theta}{\tan\theta}$
8. $\lim_{\theta\to 0} \dfrac{\tan 5\theta}{\sin 2\theta}$
9. $\lim_{\theta\to 0} \dfrac{\cot \pi\theta \sin\theta}{2\sec\theta}$
10. $\lim_{t\to 0} \dfrac{\sin^2 3t}{2t}$
11. $\lim_{t\to 0} \dfrac{\tan^2 3t}{2t}$
12. $\lim_{t\to 0} \dfrac{\tan 2t}{\sin 2t - 1}$
13. $\lim_{t\to 0} \dfrac{\sin(3t)+4t}{t\sec t}$

B.

Find the limits in Problems 61–68. The square brackets in Problems 61–64 denote the greatest integer function.

61. $\lim_{x\to 0^+} [x]$
62. $\lim_{x\to 0^-} [x]$
63. $\lim_{x\to 0.5} [x]$
64. $\lim_{x\to 2^-} \dfrac{x}{[x]}$
65. $\lim_{x\to 0^+} \dfrac{x}{|x|}$
66. $\lim_{x\to 0^-} \dfrac{x}{|x|}$
67. $\lim_{x\to 3^+} \dfrac{x^2-9}{|x-3|}$
68. $\lim_{x\to 3^-} \dfrac{x^2-9}{|x-3|}$
69. For what values of c does the greatest integer function $f(x) = [x]$ approach a limit as $x \to c$?
70. For what values of c does $f(x) = x/|x|$ approach a limit as $x \to c$?

Find the limits in Problems 71–86.

71. $\lim_{x\to 0} \dfrac{1+\sin x}{\cos x}$
72. $\lim_{x\to 0^+} \cos x$
73. $\lim_{t\to 0} \dfrac{t}{\sin t}$
74. $\lim_{h\to 0} \dfrac{\sin^2 h}{h}$
75. $\lim_{h\to 0} \dfrac{\sin^2 h}{h^2}$
76. $\lim_{t\to 0} \dfrac{2\sin t\cos t}{t}$
77. $\lim_{x\to 0} \tan x$
78. $\lim_{\theta\to 0} \dfrac{\tan\theta}{\theta}$
79. $\lim_{x\to 0^-} \sin x$
80. $\lim_{x\to 0^+} \dfrac{\sin x}{|x|}$
81. $\lim_{x\to 0^-} \dfrac{\sin x}{|x|}$
82. $\lim_{x\to 0} x\cos x$
83. $\lim_{x\to 0} \dfrac{\sin 2x}{x}$
84. $\lim_{x\to 0} \dfrac{\sin 5x}{\sin 3x}$
85. $\lim_{y\to 0} \dfrac{\tan 2y}{3y}$
86. $\lim_{x\to 0} \dfrac{\sin 2x}{2x^2+x}$

C.

Evaluate the limits in Problems 43–58, if they exist.

43. $\lim_{x\to\infty} \dfrac{\sin x}{x}$
44. $\lim_{x\to\infty} \dfrac{x+\sin x}{2x+5}$
45. $\lim_{x\to\infty} \dfrac{1+\sin x}{x}$
46. $\lim_{x\to 1} \dfrac{x^2-4}{x^3-8}$
47. $\lim_{x\to 0} \dfrac{x}{\tan 3x}$
48. $\lim_{x\to\infty} \dfrac{x\sin x}{x+\sin x}$
49. $\lim_{x\to a} \dfrac{x^2-a^2}{x-a}$
50. $\lim_{x\to a} \dfrac{x^2-a^2}{x+a}$
51. $\lim_{h\to 0} \dfrac{(x+h)^2-x^2}{h}$
52. $\lim_{h\to 0} \dfrac{\sqrt{x+h}-\sqrt{x}}{h}$
53. $\lim_{\Delta x\to 0} \dfrac{1/(x+\Delta x)-1/x}{\Delta x}$
54. $\lim_{x\to 0^+} \dfrac{1}{x}$
55. $\lim_{x\to 1} \dfrac{1-\sqrt{x}}{1-x}$
56. $\lim_{x\to 1} \dfrac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3}$
57. $\lim_{x\to\infty} (1-x\cos x)$
58. $\lim_{x\to 1} \dfrac{\sqrt{x+1}-\sqrt{2x}}{x^2-x}$

Find the limits in Problems 59–66.

59. $\lim_{x\to 0^+} \dfrac{|x|}{x}$
60. $\lim_{x\to 0^-} \dfrac{|x|}{x}$
61. $\lim_{x\to 4^-} ([x]-x)$
62. $\lim_{x\to 4^+} ([x]-x)$
63. $\lim_{x\to 3^+} \dfrac{[x]^2-9}{x^2-9}$
64. $\lim_{x\to 3^-} \dfrac{[x]^2-9}{x^2-9}$
65. $\lim_{x\to 0} x[x]$
66. $\lim_{x\to 0^+} \dfrac{\sqrt{x}}{\sqrt{4+\sqrt{x}}-2}$

D.

35. $\lim_{x\to 0^+} \dfrac{5}{2x}$
36. $\lim_{t\to 2^+} \dfrac{t^2+4}{t-2}$
37. $\lim_{t\to 2} \dfrac{t^2-4}{t-2}$
38. $\lim_{x\to 2^-} \dfrac{x}{x-2}$
39. $\lim_{x\to 1^+} \dfrac{x}{x-1}$
40. $\lim_{x\to 0} \dfrac{|x|}{|x|+1}$
41. $\lim_{x\to -1^-} \dfrac{1}{x+1}$
42. $\lim_{x\to 0} \dfrac{1}{|x|}$
43. $\lim_{x\to -2^+} \dfrac{1}{x+2}$
44. $\lim_{x\to 3^-} \dfrac{x^2}{x-3}$
45. $\lim_{x\to 3} \dfrac{x-3}{x^2}$
46. $\lim_{x\to 1^+} \dfrac{2}{x^2-1}$
47. $\lim_{x\to 2^-} \dfrac{x^2+5}{x-2}$
48. $\lim_{x\to 2} \dfrac{x-2}{x^2+5}$
49. $\lim_{x\to -5} \dfrac{x^2+3x-10}{x+5}$
50. $\lim_{x\to 1^+} \dfrac{x+4}{x^2+2x-3}$
51. Find
$$\lim \frac{x-1}{2x^2-7x+5}$$
as (a) $x\to 0$, (b) $x\to\infty$, and (c) $x\to 1$.

A. 1. 1 3. 1 5. $\frac{1}{2}$ 7. 3 9. $\frac{1}{2\pi}$ 11. 0 13. 7

B. 61. 0 63. 0 65. 1 67. 6 69. For all c not an integer 71. 1 73. 1 75. 1 77. 0 79. 0 81. −1 83. 2 85. 2/3

C. 43. 0 45. 0 47. 1/3 49. $2a$ 51. $2x$ 53. $-1/x^2$ 55. 1/2 57. Does not exist 59. 1 61. −1 63. 0 65. 0

D. 35. ∞ 37. 4 39. ∞ 41. $-\infty$ 43. ∞ 45. 0 47. $-\infty$ 49. −7 51. a) −1/5 b) 0 c) −1/3

Pg 2

E.

21. $\lim_{x\to 4^+} \dfrac{x}{x-4}$

22. $\lim_{t\to -3^+} \dfrac{t^2-9}{t+3}$

23. $\lim_{t\to 3^-} \dfrac{t^2}{9-t^2}$

24. $\lim_{x\to \sqrt[3]{5}^+} \dfrac{x^2}{5-x^3}$

25. $\lim_{x\to 5^-} \dfrac{x^2}{(x-5)(3-x)}$

26. $\lim_{\theta\to \pi^+} \dfrac{\theta^2}{\sin\theta}$

27. $\lim_{x\to 3^-} \dfrac{x^3}{x-3}$

28. $\lim_{\theta\to (\pi/2)^+} \dfrac{\pi\theta}{\cos\theta}$

29. $\lim_{x\to 3^-} \dfrac{x^2-x-6}{x-3}$

30. $\lim_{x\to 2^+} \dfrac{x^2+2x-8}{x^2-4}$

31. $\lim_{x\to 0^+} \dfrac{[\![x]\!]}{x}$

32. $\lim_{x\to 0^-} \dfrac{[\![x]\!]}{x}$

33. $\lim_{x\to 0^-} \dfrac{|x|}{x}$

34. $\lim_{x\to 0^+} \dfrac{|x|}{x}$

35. $\lim_{x\to 0^-} \dfrac{1+\cos x}{\sin x}$

36. $\lim_{x\to \infty} \dfrac{\sin x}{x}$

F.

CAS *Find the one-sided limits in Problems 59–65. Begin by plotting the function in an appropriate window. Your computer may indicate that some of these limits do not exist, but, if so, you should be able to interpret the answer as either ∞ or $-\infty$.*

59. $\lim_{x\to 3^-} \dfrac{\sin|x-3|}{x-3}$

60. $\lim_{x\to 3^-} \dfrac{\sin|x-3|}{\tan(x-3)}$

61. $\lim_{x\to 3^-} \dfrac{\cos(x-3)}{x-3}$

62. $\lim_{x\to \frac{\pi}{2}^+} \dfrac{\cos x}{x-\pi/2}$

63. $\lim_{x\to 0^+} (1+\sqrt{x})^{1/\sqrt{x}}$

64. $\lim_{x\to 0^+} (1+\sqrt{x})^{1/x}$

65. $\lim_{x\to 0^+} (1+\sqrt{x})^{x}$

G.

7. At what points is the function

$$f(x) = \begin{cases} 0, & x<0, \\ 1, & 0\le x\le 1, \\ 0, & 1<x, \end{cases}$$

continuous? (*Hint:* Graph the function.)

8. Let $f(x)$ be defined by

$$f(x) = \begin{cases} 1 & \text{for } x<0, \\ \sqrt{1-x^2} & \text{for } 0\le x\le 1, \\ x-1 & \text{for } x>1. \end{cases}$$

Is f continuous? (*Hint:* Graph the function.)

Find the points (if any) at which the functions in Problems 12–21 are *not* continuous.

12. $y = \dfrac{1}{x-2}$

13. $y = \dfrac{1}{(x+2)^2}$

14. $y = \dfrac{x}{x+1}$

15. $y = \dfrac{x+1}{x^2-4x+3}$

16. $y = |x-1|$

17. $y = \dfrac{x+3}{x^2-3x-10}$

18. $y = \dfrac{x^3-1}{x^2-1}$

19. $y = \dfrac{1}{x^2+1}$

20. $y = \dfrac{\cos x}{x}$

21. $y = \dfrac{|x|}{x}$

22. The function $f(x)$ is defined by $f(x) = (x^2-1)/(x-1)$ when $x \ne 1$ and by $f(1) = 2$. Is f continuous at $x = 1$? Explain.

23. Define $g(3)$ in a way that extends $g(x) = (x^2-9)/(x-3)$ to be continuous at $x = 3$.

24. Define $h(2)$ in a way that extends $h(x) = (x^2+3x-10)/(x-2)$ to be continuous at $x = 2$.

25. Define $f(1)$ in a way that extends $f(x) = (x^3-1)/(x^2-1)$ to be continuous at $x = 1$.

26. Define $g(4)$ in a way that extends $g(x) = (x^2-16)/(x^2-3x+4)$ to be continuous at $x = 4$.

27. a) Graph the function

$$f(x) = \begin{cases} x, & 0\le x\le 1, \\ 2-x, & 1<x\le 2. \end{cases}$$

b) Is f continuous at $x = 1$?

Special Problem

(Famous Problem) Let $f(x) = 0$ if x is irrational and let $f(x) = 1/q$ if x is the rational number p/q in reduced form ($q > 0$).

(a) Sketch (as best you can) the graph of f on $(0, 1)$.

(b) Show that f is continuous at each irrational number in $(0, 1)$, but is discontinuous at each rational number in $(0, 1)$.

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21. ∞ **23.** ∞

25. ∞ **27.** $-\infty$ **29.** 5 **31.** 0 **33.** -1 **35.** $-\infty$

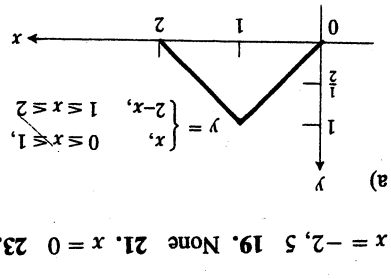
59. -1 **61.** $-\infty$ **63.** e **65.** 1

7. All x except 0, 1 **9.** All x in domain except 1, 2

11. All x except -1, 0, 1 **13.** $x = -2$ **15.** $x = 1, 3$

17. $x = -2, 5$ **19.** None **21.** $x = 0$ **23.** 6 **25.** 3/2

27. a)



b) Yes

MORE LIMIT PRACTICE WS

Pg 3

EXERCISE SET 2.6 Graphing Utility

H.

In Exercises 1–10, find the discontinuities, if any.

1. $f(x) = \sin(x^2 - 2)$
2. $f(x) = \cos\left(\dfrac{x}{x-\pi}\right)$
3. $f(x) = \cot x$
4. $f(x) = \sec x$
5. $f(x) = \csc x$
6. $f(x) = \dfrac{1}{1+\sin^2 x}$
7. $f(x) = |\cos x|$
8. $f(x) = \sqrt{2+\tan^2 x}$
9. $f(x) = \dfrac{1}{1-2\sin x}$
10. $f(x) = \dfrac{3}{5+2\cos x}$

11. Use Theorem 2.5.6 to show that the following functions are continuous everywhere by expressing them as compositions of simpler functions that are known to be continuous.
   (a) $\sin(x^3 + 7x + 1)$ (b) $|\sin x|$
   (c) $\cos^3(x+1)$ (d) $\sqrt{3+\sin 2x}$
   (e) $\sin(\sin x)$ (f) $\cos^5 x - 2\cos^3 x + 1$

12. (a) Prove that if $g(x)$ is continuous everywhere, then so are $\sin(g(x))$, $\cos(g(x))$, $g(\sin(x))$, and $g(\cos(x))$.
   (b) Illustrate the result in part (a) with some of your own choices for g.

Find the limits in Exercises 13–35.

13. $\lim\limits_{x\to+\infty} \cos\left(\dfrac{1}{x}\right)$
14. $\lim\limits_{x\to+\infty} \sin\left(\dfrac{2}{x}\right)$
15. $\lim\limits_{x\to+\infty} \sin\left(\dfrac{\pi x}{2-3x}\right)$
16. $\lim\limits_{h\to 0} \dfrac{\sin h}{2h}$
17. $\lim\limits_{\theta\to 0} \dfrac{\sin 3\theta}{\theta}$
18. $\lim\limits_{\theta\to 0^+} \dfrac{\sin\theta}{\theta^2}$
19. $\lim\limits_{x\to 0^-} \dfrac{\sin x}{|x|}$
20. $\lim\limits_{x\to 0} \dfrac{\sin^2 x}{3x^2}$
21. $\lim\limits_{x\to 0^+} \dfrac{\sin x}{5\sqrt{x}}$
22. $\lim\limits_{x\to 0} \dfrac{\sin 6x}{\sin 8x}$
23. $\lim\limits_{x\to 0} \dfrac{\tan 7x}{\sin 3x}$
24. $\lim\limits_{\theta\to 0} \dfrac{\sin^2\theta}{\theta}$
25. $\lim\limits_{h\to 0} \dfrac{h}{\tan h}$
26. $\lim\limits_{h\to 0} \dfrac{\sin h}{1-\cos h}$
27. $\lim\limits_{\theta\to 0} \dfrac{\theta^2}{1-\cos\theta}$
28. $\lim\limits_{x\to 0} \dfrac{x}{\cos\left(\frac{1}{2}\pi - x\right)}$
29. $\lim\limits_{\theta\to 0} \dfrac{\theta}{\cos\theta}$
30. $\lim\limits_{t\to 0} \dfrac{t^2}{1-\cos^2 t}$
31. $\lim\limits_{h\to 0} \dfrac{1-\cos 5h}{\cos 7h - 1}$
32. $\lim\limits_{x\to 0^+} \sin\left(\dfrac{1}{x}\right)$
33. $\lim\limits_{x\to 0^+} \cos\left(\dfrac{1}{x}\right)$
34. $\lim\limits_{x\to 0} \dfrac{x^2 - 3\sin x}{x}$
35. $\lim\limits_{x\to 0} \dfrac{2x+\sin x}{x}$

In Exercises 36–39: (i) Construct a table to estimate the limit by evaluating the function near the limiting value. (ii) Find the exact value of the limit.

36. $\lim\limits_{x\to 5} \dfrac{\sin(x-5)}{x^2-25}$
37. $\lim\limits_{x\to 2} \dfrac{\sin(2x-4)}{x^2-4}$
38. $\lim\limits_{x\to -2} \dfrac{\sin(x^2+3x+2)}{x+2}$
39. $\lim\limits_{x\to -1} \dfrac{\sin(x^2+3x+2)}{x^3+1}$

40. Find a value for the constant k that makes

$$f(x) = \begin{cases} \dfrac{\sin 3x}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$$

continuous at $x = 0$.

41. Find a nonzero value for the constant k that makes

$$f(x) = \begin{cases} \dfrac{\tan kx}{x}, & x < 0 \\ 3x + 2k^2, & x \geq 0 \end{cases}$$

continuous at $x = 0$.

42. Is

$$f(x) = \begin{cases} \dfrac{\sin x}{|x|}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

continuous at $x = 0$?

43. In each part, find the limit by making the indicated substitution.
   (a) $\lim\limits_{x\to+\infty} x\sin\dfrac{1}{x}$; $t = \dfrac{1}{x}$
   (b) $\lim\limits_{x\to-\infty} x\left(1-\cos\dfrac{1}{x}\right)$; $t = \dfrac{1}{x}$
   (c) $\lim\limits_{x\to\pi} \dfrac{\pi - x}{\sin x}$; $t = \pi - x$

44. Find $\lim\limits_{x\to 2} \dfrac{\cos(\pi/x)}{x-2}$. [*Hint:* Let $t = \dfrac{\pi}{2} - \dfrac{\pi}{x}$.]

45. Find $\lim\limits_{x\to 1} \dfrac{\sin(\pi x)}{x-1}$.

46. Find $\lim\limits_{x\to\pi/4} \dfrac{\tan x - 1}{x - \pi/4}$.

▶ **Exercise Set 2.6 (Page 163)**

1. none 3. $x = n\pi, n = 0, \pm 1, \pm 2, \ldots$
5. $x = n\pi, n = 0, \pm 1, \pm 2, \ldots$ 7. none
9. $2n\pi + (\pi/6), 2n\pi + (5\pi/6), n = 0, \pm 1, \pm 2, \ldots$
11. (a) $\sin x, x^3 + 7x + 1$ (b) $|x|, \sin x$ (c) $x^3, \cos x, x + 1$
(d) $\sqrt{x}, 3 + x, \sin x, 2x$ (e) $\sin x, \sin x$ (f) $x^5 - 2x^3 + 1, \cos x$
13. 1 15. $-\sqrt{3}/2$ 17. 3 19. -1 21. 0 23. $\frac{7}{3}$
25. 1 27. 2 29. 0 31. $-\frac{25}{49}$ 33. does not exist
35. 3 37. $\frac{1}{2}$ 39. $\frac{1}{3}$ 41. $k = \frac{1}{2}$ 43. (a) 1 (b) 0 (c) 1
45. $-\pi$ 47. $-|x| \leq x\cos(50\pi/x) \leq |x|$

HOLT 2.3 & 2.4

2.3

23. $\lim_{x\to 3} \dfrac{x^2 - x - 6}{x^3 - 3x^2 + x - 3}$

24. $\lim_{x\to 4} \dfrac{x^3 + 4x^2 + x + 4}{x^2 + 3x - 4}$

25. $\lim_{x\to 100} \dfrac{x - 100}{\sqrt{x} - 10}$

26. $\lim_{x\to 1/16} \dfrac{x^{1/2} - 1/4}{x^{1/4} - 1/2}$

27. $\lim_{y\to 1/27} \dfrac{y^{2/3} - 1/9}{y^{1/3} - 1/3}$

28. $\lim_{y\to 2} \dfrac{\sqrt{y} - \sqrt{2}}{y^2 - 2y}$

29. $\lim_{y\to 1/2} \dfrac{6y - 3}{y(1 - 2y)}$

30. $\lim_{x\to 0} x\left(1 - \dfrac{1}{x}\right)$

31. $\lim_{x\to -2} \left(\dfrac{x^2}{x + 2} - \dfrac{4}{x + 2}\right)$

32. $\lim_{x\to 0} \dfrac{1 + 1/x}{2 + 1/x}$

33. $\lim_{x\to 0} \dfrac{e^{2x} - 1}{e^x - 1}$

34. $\lim_{x\to 0} \dfrac{e^{3x} - 1}{e^x - 1}$

2.4

EXERCISES 2.4

In Exercises 1–30 use the results of this section to evaluate the limit.

1. $\lim_{x\to \pi/3} (\sqrt{3}\sin x - 2x)$

2. $\lim_{x\to 0} \dfrac{x^2 - 2}{\cos x}$

3. $\lim_{x\to -\pi/3} 3x^2 \cos x$

4. $\lim_{x\to 0} \dfrac{1 - \cos x}{1 - x}$

5. $\lim_{x\to 0} e^x \cos x$

6. $\lim_{x\to e} \dfrac{\ln x}{e^{-x}}$

7. $\lim_{y\to 2\pi/3} \dfrac{\pi \sin y \cos y}{y}$

8. $\lim_{y\to 0} \dfrac{\pi \sin y \cos y}{y}$

9. $\lim_{x\to 3} \sqrt{3x^3}$

10. $\lim_{x\to \sqrt{5}} (9 - x^2)^{-5/2}$

11. $\lim_{t\to 3\pi/2} \sin\left(\dfrac{\pi}{2}\sin t\right)$

12. $\lim_{x\to \pi/6} e^{\sin x}$

13. $\lim_{x\to 1/2} \ln(6x^2 - 1)$

14. $\lim_{x\to e} \dfrac{\ln(\ln x)}{\ln x}$

15. $\lim_{x\to 0} \ln\left(\dfrac{e^x - 1}{x}\right)$

16. $\lim_{x\to 0} \dfrac{e^{2x} - 1}{x}$

17. $\lim_{x\to 0} \dfrac{\sin 3x}{5x}$

18. $\lim_{x\to 0} \dfrac{\cos 4x - 1}{x}$

19. $\lim_{x\to 0} \dfrac{\sin x^{1/3}}{x^{1/3}}$

20. $\lim_{x\to 0} \dfrac{\cos x^2 - 1}{x^2}$

21. $\lim_{t\to 0} \dfrac{\cos^2 t - 1}{t}$

22. $\lim_{t\to 0} \dfrac{\cos t - 1}{\sqrt[3]{t}}$

23. $\lim_{y\to 0} \dfrac{\tan y}{y}$

24. $\lim_{y\to 0} y \cot y$

25. $\lim_{x\to 0} \dfrac{\sin x}{\sin 2x}$

26. $\lim_{x\to 0} \dfrac{\sin^2 x}{1 - \cos x}$

27. $\lim_{x\to \pi} \dfrac{\tan^2 x}{1 + \sec x}$

28. $\lim_{x\to 0} \dfrac{1 - \cos 3x}{\sin 3x}$

29. $\lim_{x\to 0} \dfrac{2x - \cot x}{x + 3\cot x}$

30. $\lim_{x\to 0} \dfrac{\cos x - \sec x}{1 - \sec x}$

Answers

2.3

25. 20 **31.** -4

35. $y + 2 = 2(x + 1)$

39. $y - \frac{1}{2} = -\frac{1}{4}(x + 1)$

43. continuous at 2

47. continuous at 4

49. a. 6 **b.** 1 **c.** impossible

57. a. No such c exists. **b.** all $c > \frac{9}{8}$

61. a. Product Rule not applicable **b.** 1

23. $\frac{1}{2}$ **29.** -6

21. -8 **27.** $\frac{2}{3}$ **33.** 2

37. $y - \frac{1}{2} = -\frac{1}{4}(x - 2)$

41. $y - \frac{1}{4} = \frac{1}{8}(x - 16)$

45. continuous at 0

2.4

Section 2.4

1. $\frac{3}{2} - \frac{2}{3}\pi$ **3.** $\frac{1}{6}\pi^2$ **5.** 1 **7.** $-\frac{3}{8}\sqrt{3}$

9. 9 **11.** -1 **13.** $\ln\frac{1}{2}$ **15.** 0

17. $\frac{3}{5}$ **19.** 1 **21.** 0 **23.** 1

25. $\frac{1}{2}$ **27.** -2 **29.** $\frac{1}{3}$ **31.** 0