Name: ____________________________________ Date: _________________

SOLVING EXPONENTIAL EQUATIONS WITH LOGARITHMS
ALGEBRA 2 WITH TRIGONOMETRY

Earlier in this unit, we used the **Method of Common Bases** to solve exponential equations. This technique is quite limited, however, because it requires the two sides of the equation to be expressed using the same base. A more general method utilizes our calculators and the third logarithm law:

THE THIRD LOGARITHM LAW

$$\log_b\left(a^x\right) = x\log_b a$$

***Exercise* #1:** Solve: $4^x = 8$ using (a) common bases and (b) the logarithm law shown above.

(a) Method of Common Bases

(b) Logarithm Approach

The beauty of this logarithm law is that it removes the variable from the exponent. This law, in combination with the logarithm base 10, the **common log**, allows us to solve almost any exponential equation.

***Exercise* #2:** Solve each of the following equations for the value of *x*. Round your answers to the nearest *hundredth*.

(a) $5^x = 18$

(b) $4^x = 100$

(c) $2^x = 1560$

These equations can become more complicated, but each and every time we will use the logarithm law to transform an exponential equation into one that is more familiar (linear, quadratic, etc).

***Exercise* #3:** Solve each of the following equations for *x*. Round your answers to the nearest *hundredth*.

(a) $6^{x+3} = 50$

(b) $(1.03)^{\frac{x}{2}-5} = 2$

Now that we are familiar with this method, we can revisit some of our exponential models from earlier in the unit. Recall that for an exponential function that is growing:

> If quantity Q is known to increase by a fixed percentage p, in decimal form, then Q can be modeled by
>
> $$Q(t) = Q_0(1+p)^t$$
>
> where Q_0 represents the amount of Q present at $t = 0$ and t represents time.

***Exercise* #4:** A biologist is modeling the population of bats on a tropical island. When he first starts observing them, there are 104 bats. The biologist believes that the bat population is growing at a rate of 3% per year.

(a) Write an equation for the number of bats, $B(t)$, as a function of the number of years, t, since the biologist started observing them.

(b) Using your equation from part (a), algebraically determine the number of years it will take for the bat population to reach 200. Round your answer to the nearest year.

***Exercise* #5:** A stock has been declining in price at a steady pace of 5% per week. If the stock started at a price of $22.50 per share, determine algebraically the number of weeks it will take for the price to reach $10.00. Round your answer to the nearest week.

As a final discussion, we return to evaluating logarithms using our calculator. Since the calculator has only the base 10 logarithm (and one other to be named later), it is limited in evaluating logs.

***Exercise* #6:** Consider the expression $\log_5 70$.

(a) Write an equivalent exponential equation for the equation $\log_5 70 = x$.

(b) Solve this equation for x, thus evaluating $\log_5 70$. Round your answer to the nearest *hundredth*.

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SOLVING EXPONENTIAL EQUATIONS WITH LOGARITHMS
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

SKILLS

1. Which of the following values, to the nearest *hundredth*, solves: $7^x = 500$?

(1) 3.19 (3) 2.74

(2) 3.83 (4) 2.17

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2. The solution to $2^{x/3} = 52$, to the nearest *tenth*, is which of the following?

(1) 7.3 (3) 11.4

(2) 9.1 (4) 17.1

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3. To the nearest *hundredth*, the value of *x* that solves $5^{x-4} = 275$ is

(1) 6.73 (3) 8.17

(2) 5.74 (4) 7.49

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4. Solve each of the following exponential equations. Round each of your answers to the nearest *hundredth*.

(a) $9^{x-3} = 250$ (b) $50(2)^x = 1000$ (c) $5^{x/10} = 35$

5. Solve each of the following exponential equations. Be careful with your use of parentheses. Express each answer to the nearest *hundredth*.

(a) $6^{2x-5} = 300$ (b) $\left(1/2\right)^{\frac{x}{3}+1} = 1/6$ (c) $500(1.02)^{\frac{x}{12}} = 2300$

6. Solve each of the following trigonometric equations for all values of x on the interval $0^\circ \le x \le 360^\circ$. Round all answers to the nearest *tenth* of a degree.

(a) $3^{\cos x} = 2$

(b) $5^{\sin x} = \frac{1}{4}$

APPLICATIONS

7. The population of Charleston is growing at a rate of 3.5% per year. If its current population is 12,500, in how many years will the population exceed 20,000? Round your answer to the nearest year. Only an *algebraic* solution is acceptable.

8. A radioactive substance is decaying such that 2% of its mass is lost every year. Originally there were 50 kilograms of the substance present.

(a) Write an equation for the amount, A, of the substance left after t-years.

(b) Find the amount of time that it takes for only half of the initial amount to remain. Round your answer to the nearest tenth of a year.

REASONING

9. By following a similar procedure as in *Exercise* #6, find a express $y = \log_b x$ equivalently in terms of $\log b$ and $\log x$. This is known as the **Change of Base Formula**.