Name: _____ Date: _____

THE WINDING FUNCTION
ALGEBRA 2 HONORS

Most functions in mathematics are what are known as **algebraic**, i.e. the outputs can be determined from the inputs by arithmetic. Today we will see the first non-algebraic function, and one that will be exceptionally important to us in trigonometry, the **winding function**. It is defined below:

THE WINDING FUNCTION

The input of the winding function is an angle, let's call it θ, and the output of the winding function is:

$w\ \theta\ =$ coordinate pair from the unit circle

This is a strange function, indeed, because you must look at a geometric object, **the unit circle**, to determine the output, which isn't a number but a pair of numbers. For the winding function, we are literally **winding a number line around the unit circle an infinite number of times**.

***Exercise* #1:** State the value of the winding function for each of the following:

(a) $w\ 90^\circ\ =$

(b) $w\ 30^\circ\ =$

(c) $w\ 45^\circ\ =$

(d) $w\left(\frac{\pi}{3}\right) =$

(e) $w\left(\frac{3\pi}{2}\right) =$

(f) $w\ 2\pi\ =$

Although it might be difficult, we must find a way to produce the values of the winding function for any of the **special angles**, i.e. multiples of 30°, 45°, or 60°. The key here will be the use of **reference angles**. One example should suffice.

***Exercise* #2:** Consider the angle $\theta = 150^\circ$.

(a) What is this angle's reference angle?

(b) State the winding function values for both this angle and its reference.

We see that the angle and its reference will have the same winding value outputs, just different by sign.

***Exercise* #3:** State the winding function values for each of the following angles. First, determine the reference angle for each and use this as a guide.

(a) $w\ 120^\circ\ =$ (b) $w\ 330^\circ\ =$ (c) $w\ 225^\circ\ =$

(d) $w\left(\frac{7\pi}{4}\right)=$ (e) $w\left(\frac{5\pi}{6}\right)=$ (f) $w\left(\frac{4\pi}{3}\right)=$

Since the angle is not restricted to the interval $\left[0^\circ, 360^\circ\right]$, we should also be able to evaluate the winding function for angles outside of this interval (whether we are in degrees or radians). The key here will be both reference angles and **coterminal angles** (which will have the same winding function outputs).

***Exercise* #4:** State the winding function values for each of the following angles.

(a) $w\ 750^\circ\ =$ (b) $w\ -960^\circ\ =$ (c) $w\ 405^\circ\ =$

(d) $w\left(\frac{11\pi}{3}\right)=$ (e) $w\left(\frac{11\pi}{2}\right)=$ (f) $w\left(-\frac{19\pi}{6}\right)$

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THE WINDING FUNCTION
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1. Give the outputs for each of the following winding functions where the inputs have been given in units of degrees.

   (a) $w\ 60^\circ =$ (b) $w\ 210^\circ =$ (c) $w\ 315^\circ =$

   (d) $w\ 120^\circ =$ (e) $w\ 90^\circ =$ (f) $w\ 135^\circ =$

2. Give the outputs for each of the following winding functions where the inputs have been given in units of degrees.

   (a) $w\ 450^\circ =$ (b) $w\ -120^\circ =$ (c) $w\ -750^\circ =$

   (d) $w\ 1830^\circ =$ (e) $w\ -1305^\circ =$ (f) $w\ 900^\circ =$

3. Give the outputs for each of the following winding functions where the inputs have been given in units of radians.

   (a) $w\left(\frac{5\pi}{3}\right) =$ (b) $w\left(\frac{3\pi}{2}\right) =$ (c) $w\left(\frac{\pi}{6}\right) =$

   (d) $w\left(\frac{3\pi}{4}\right) =$ (e) $w\left(\frac{7\pi}{6}\right) =$ (f) $w\left(\frac{4\pi}{3}\right) =$

4. Give the outputs for each of the following winding functions where the inputs have been given in units of radians.

(a) $w\left(\frac{11\pi}{4}\right) =$ (b) $w\left(\frac{17\pi}{3}\right) =$ (c) $w\left(-\frac{9\pi}{2}\right) =$

(d) $w\left(\frac{13\pi}{6}\right) =$ (e) $w\ 13\pi\ =$ (f) $w\ 22\pi\ =$

5. What is the largest value that either coordinate in the output to the winding function can have? What is the smallest coordinate? Explain why.

6. Give four angles, in degrees, that could solve the equation $w\ \theta\ =\ 0,1$. Include at least one negative angle.

7. If an angle, θ, had its terminal ray in the second quadrant and $w\ \theta\ = \left(a, \frac{\sqrt{3}}{5}\right)$ then determine the value of a in exact form.